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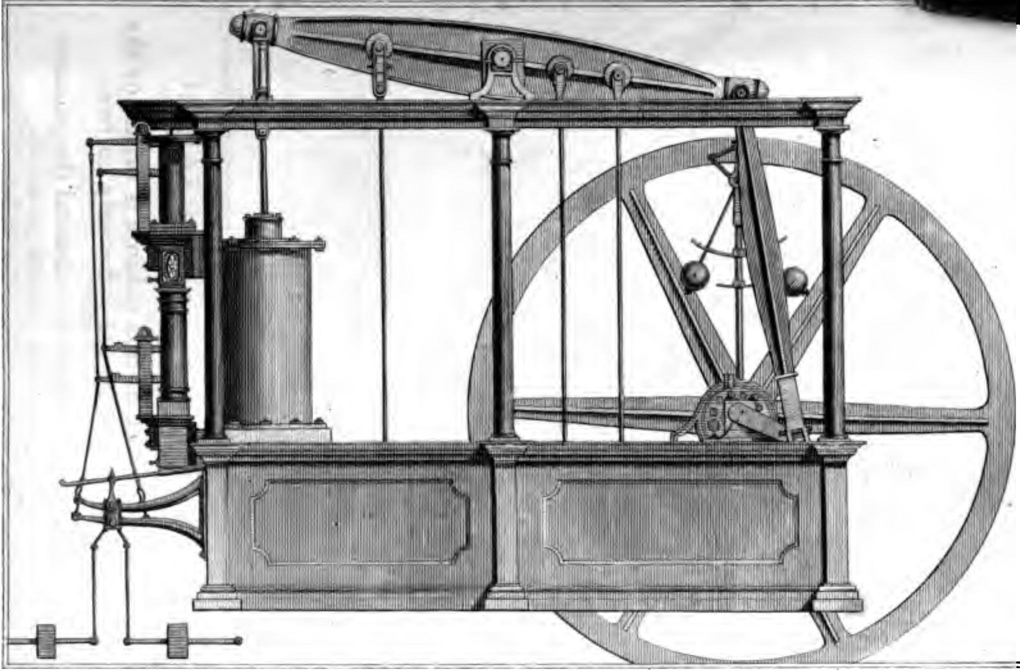

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COMPENDIUM OF MECHANICS;

OR

TEXT BOOK

FOR

ENGINEERS, MILL-WRIGHTS, MACHINE-MAKERS,
FOUNDERS, SMITHS, &c.

CONTAINING

PRACTICAL RULES AND TABLES

CONNECTED WITH THE

STEAM ENGINE, WATER WHEEL, FORCE PUMP,
AND MECHANICS IN GENERAL:

ALSO,

Examples for each Rule,

CALCULATED IN COMMON DECIMAL ARITHMETIC,
Which renders this Treatise particularly adapted for the use of
OPERATIVE MECHANICS.

By ROBERT BRUNTON.

WITH PLATES.

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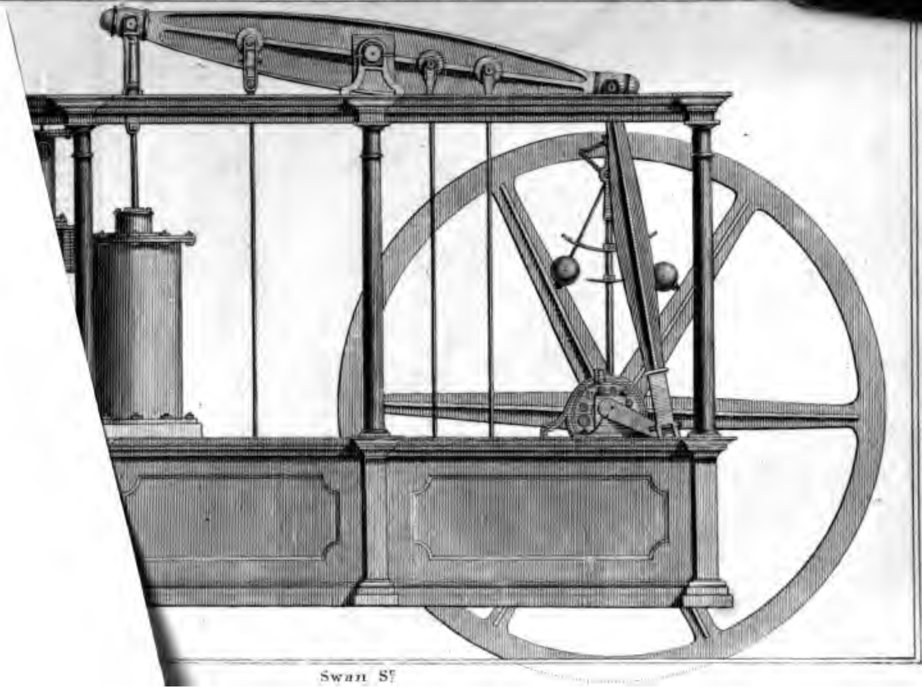

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SECOND EDITION, IMPROVED AND ENLARGED.

GLASGOW:
JOHN NIVEN, JUN., 118, Trongate;
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1825.

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1825

TO THE

MECHANICS OF GLASGOW,

THIS COMPENDIUM

OF

PRACTICAL MECHANICS,

IS INSCRIBED BY

THEIR OBEDIENT SERVANT,

Robert Brunton.

PREFACE.

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Most of the Rules and Tables have been selected from the latest eminent Publications on these subjects, and information procured from every possible source, with a view of rendering this Work useful for practical purposes.

The want of a Text Book for Operative Mechanics has been long felt.—The great inconvenience arising from this, was the cause of the Compiler collecting the following Rules for his own personal use:—and having, with several other Mechanics, experienced the great advantage derived from these Memoranda, is induced to submit them to the Public, trusting they will be found to contain much useful information.

GLASGOW,
February, 1824.

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It is a source of much gratification to an author, to know that his work has accomplished the end for which it was intended. This pleasure has been experienced by the Compiler of the following little Work. In consequence of the favourable reception it has received, he has been induced to give it a careful revisal; and from the alterations and additions that have been made, he trusts it will now be found still more useful to the operative mechanic.

LONDON, }
April, 1825. }

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EXPLANATIONS

OF THE

Characters used in the following Calculations.

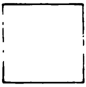
$+$ Signifies Addition, as $5 + 3$ is 8.
$-$ ——— Subtraction, as $5 - 3$ is 2.
$\times$ ——— Multiplication, as 5×3 is 15.
$\div$ ——— Division, as $15 \div 3$ is 5, or $\frac{15}{3}$ is 5.
$: :: :$ ——— Proportion, as 2 is to 3, as 4 is to 6.
$=$ ——— Equality, as $5 + 3 = 8$.
$\sqrt{\ }$ ——— Square Root, $\sqrt{9} = 3$.
$\sqrt[3]{\ }$ ——— Cube Root, as $\sqrt[3]{27} = 3$.

3^2 Signifies that 3 is to be squared as $3^2 = 9$.
3^3 ——— 3 is to be cubed as $3^3 = 27$.
The Bar signifies that 2 numbers are to be taken together, as $3 \times \overline{5 + 3} = 24$.

MENSURATION.

Plate 1st

1



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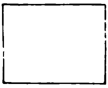

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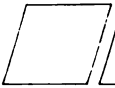

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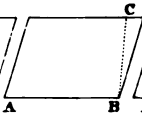

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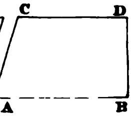

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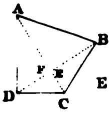

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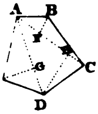

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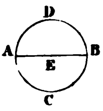

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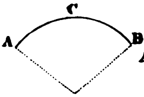

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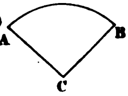

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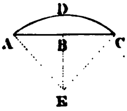

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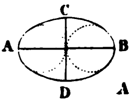

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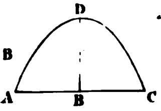

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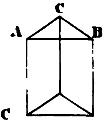

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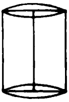

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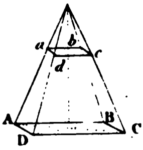

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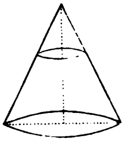

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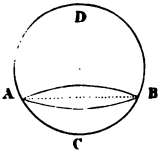


Explanation of Plate First.

No. 1. A Square.
2. A Rectangle.
3. A Rhombus.
4. A Rhomboid.
5. A Trapezoid.
6. A Trapezium.
7. An Irregular Polygon.
8. A Circle, A E the radius, A E B the diameter, A C B D the circumference.
9. An Arc of a Circle, A C B.
10. A Sector of a Circle, A C B.
11. A Segment of a Circle, A B C D.
12. An Ellipsis or Oval, A B the long diameter, C D the short diameter.
13. A Parabola, A B C the base, B D the perpendicular height.
14. A Prism, A B C the perimeter; or the circumference of the end of a cylinder, is the perimeter of that cylinder.
15. A Cylinder.
16. A Pyramid, A B C D the base; A B C D *a b c d* the frustum.
17. A Cone.
18. A Sphere, A B C D the circumference, A B C a segment.

Explanation of Plate Second.

A—Lever of the first order.

B—Lever of the second order.

C—Lever of the third order.

D—Bended Lever: the effective power and weight on a bended lever, is as the distance between the points of action and the fulcrum, as *a b c b*. The distance being taken at right angles to the direction of the forces.

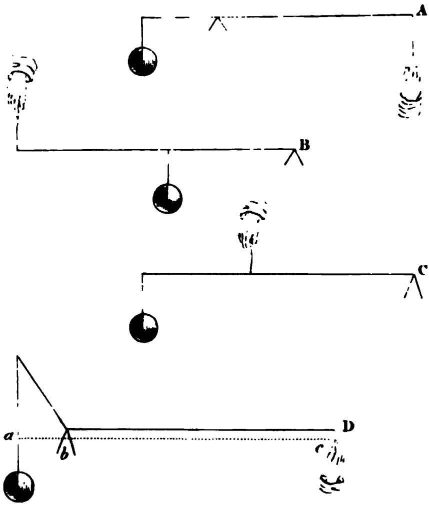
Fig. 1. A Diagram, explanatory of the Wheel and Axle.

— 2. A Diagram, explanatory of the Pulley, when the directions of the cords are not parallel.

— 3. A Diagram, explanatory of the inclined plane, when the power is not in a direction with the plane.

MECHANICAL POWERS.

Plate 2nd



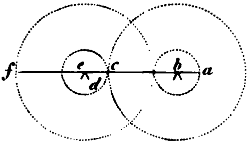


Fig. 1.

Fig. 2.

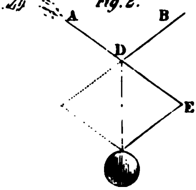
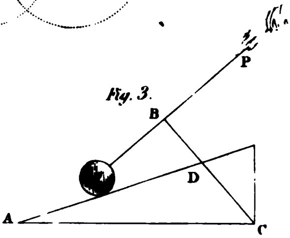
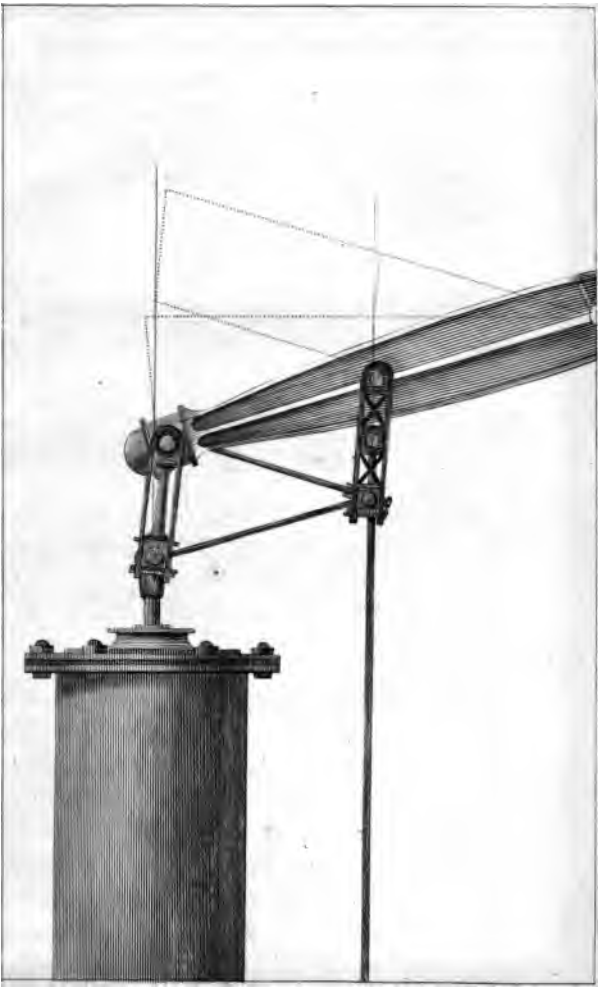


Fig. 3.



PARALLEL MOTION.

Plate. 3^{d}



Swan

COMPENDIUM

OF

MECHANICS.

WEIGHTS AND MEASURES.

TROY WEIGHT.

					Mark.
Gr.	*Dwt.*			Grains, ...	*Gr.*
24 =	1	*Oz.*		Penny-weights, . .	*Dwt.*
480 =	20 =	1	*Lib.*	Ounce, ...	*Oz.*
5760 =	240 =	12 =	1	Pound, ...	*Lib.*

By this Weight are weighed Gold, Silver, and Jewels.

AVOIRDUPOIS WEIGHT.

							Mark.
Dr.	*Oz.*					Dram, . .	*Dr.*
16=	1	*Lib.*				Ounce, .	*Oz.*
256=	16=	1	*Qr.*			Pound, . .	*Lib.*
7168=	448=	28=	1	*Cwt.*		Quarter, .	*Qr.*
28672=	1792=	112=	4=	1	*Ton*	Hund. wt. .	*Cwt.*
573440=	35840=	2240=	80=	20=	1	Ton, . .	*Ton*

By this Weight all Metals, except Gold and Silver, are weighed.

AVOIRDUPOIS WEIGHT.

			Oz.	Dwt.	Qr.	
Note.	1 Lib. Avoir.	=	14 .	11 .	15½	Troy.
	1 Oz. do.	=		18 .	5½	do.
	1 Dr. do.	=		1 .	3½	do.

LONG MEASURE.

							Mark.
In.	*Ft.*					Inch, . .	*In.*
12=	1	*Yd.*				Foot, . .	*Ft.*
36=	3 =	1	*Pl.*			Yard, . .	*Yd.*
198=	16½=	5½=	1	*Fur.*		Pole or Rod,	*Pl.*
7920=	660 =	220 =	40=	1	*M.*	Furlong, .	*Fur.*
63360=	5280 =	1760 =	320=	8=	1	Mile, . .	*M.*

3 Miles = 1 League, marked Lea.

$69\frac{1}{11}$ Miles nearly = 1 Degree, marked °

SQUARE MEASURE.

							Mark.
Sq. In.	*Sq. Ft.*					Square Inch,	*Sq. In.*
144=	1	*Sq. Yd.*				——— Foot,	*Sq. Ft.*
1296=	9 =	1	*Sq. Pl.*			——— Yard,	*Sq. Yd.*
39204=	272¼=	30¼=	1	*Rd.*		——— Pole,	*Sq. Pl.*
1568160=	10890 =	1210 =	40=	1	*Acr.*	Rood, . .	*Rd.*
6272640=	43560 =	4840 =	160=	4=	1	Acre, . .	*Acr.*

1089 Scotch Acres = 1369 English Acres.

DRY MEASURE.

Pts. *Gal.*
8= 1 *Pec.*
16= 2= 1 *Bu.*
64= 8= 4= 1 *Qr.*
512= 64= 32= 8= 1 *Wey*
2560=320=160=40= 5=1 *Last*
5120=640=320=80=10=2=1

	Mark.
Pints, . .	*Pts.*
Gallon, . .	*Gal.*
Peck, . .	*Pec.*
Bushel, . .	*Bu.*
Quarter, . .	*Qr.*
Wey, Load, or Ton,	*Wey*
Last, . . .	*Last*

A Chaldron of Coals in London = 36 Bushels, and weighs 3136 libs. Avoirdupois, or 1 Ton 8 Cwt. nearly.

ALE MEASURE.

Pts. *Qt.*
2= 1 *Gal.*
8= 4= 1 *Bar.*
288=144= 36=1 *Hhd.*
432=216= 54=1½= 1 *Butt*
864=432=108=3 = 2 = 1 *Tun*
1728=864=216=6 = 4 = 2 = 1

	Mark.
Pints, . .	*Pts.*
Quart, .	*Qt.*
Gallon, .	*Gal.*
Barrel, .	*Bar.*
Hogshead,	*Hhd.*
Butt, . .	*Butt*
Tun, . .	*Tun*

Note. The Ale Gallon contains 282 Cubic or Solid Inches.

WINE MEASURE.

pts	*qt*	*gal*	*tier*	*hhd*	*pun*	*pipe*	*tun*
2	1						
8	4	1					
336	168	42	1				
504	252	63	1½	1			
672	336	84	2	1⅓	1		
1008	504	126	3	2	1½	1	
2016	1008	252	6	4	3	2	1

	Mark.
Pints, . .	*pts*
Quart, .	*qt*
Gallon, .	*gal*
Tierce, .	*tier*
Hogshead,	*hhd*
Puncheon,	*pun*
Pipe or Butt,	*pipe*
Tun, . .	*tun*

Note. The Wine Gallon contains 231 Cubic or Solid Inches; and it is remarkable, that the Wine Gallon is to the Ale Gallon, nearly as the Pound Troy is to the Pound Avoirdupois.

SOLID MEASURE.

Cubic Inches	*Cub. Ft.*	*Cub. Yd.*	*Fathom*
1728	1		
15552	9	1	
373248	216	8	1

In taking the solid contents of any Mass, there is seldom any other Measure than the Cubic Foot used.

*The old and new French Weights and Measures, reduced to the English Standard.**

The Paris pound, *poids de marc of Charlemagne*, contains 9216 Paris grains; it is divided into 16 ounces, each ounce into 8 gros, (or drams,) and each gros into 72 grains; it is equal to 7561 English troy grains.

The English troy pound of 12 ounces, contains 5760 English troy grains, and is equal to 7021 Paris grains.

The English avoirdupois pound of 16 ounces, contains 7000 English troy grains, and is equal to 8588 Paris grains.

To reduce Paris grains to English troy grains, divide by 1.2189.

To reduce Paris ounces to English troy, divide by 1.015734; or the conversion may be made by means of the following Tables.

I. To reduce French to English Troy Weight.

			English Troy Grains.
The Paris	Pound	=	7561
	Ounce	=	472.5625
	Gros	=	59.0703
	Grain	=	.8204

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* See the Technical Repository, Vol. 3, No. 6, for June 1823.

II. To reduce Paris Long Measure to English.

		Eng. Inches.
The Paris Royal Foot of 12 Inches	=	12.7977
The Inch	=	1.0659
The Line, or one-twelfth of an Inch	=	.0074

III. To reduce French Cubic Measure to English.

		Eng. Cubical Feet.
The Paris Cubic Foot . .	=	1.211273
The Cubic Inch	=	.000700

IV. Measure of Capacity.

The Paris pint contains 58.145 English cubical inches, and the English Wine pint contains 28.875 cubical inches; or the Paris pint contains 2.0171082 English pints; therefore, to reduce the Paris pint to the English, multiply by 2.0171082.

TABLE *of the New French Weights and Measures, reduced to the English Standard.*

The French *metre*, according to the *Journal de Physique, An. 7. Prair, & Fruct*, is equal to 3 feet, 11.296 lines French, and the *gramme* to 18.827 grains. The *metre* is the ten-millionth part of the distance from the Pole to the Equator. The *gramme* is the weight of a cubic centimetre of water. The French *toise* was 76.734 inches English; and 576 French grains were equal to 472.5 English.——*See Phil. Transact.* vol. 58, p. 326.

MEASURES OF LENGTH.

		English Inches.
Millimetre	=	.03937
Centimetre	=	.39370
Decimetre	=	3.93702
Metre	=	39.37023
Decametre	=	393.70226
Hecatometre	=	3937.02260
Chiliometre	=	39370.22601
Myriometre	=	393702.26014

		M.	P.	Y.	Ft.	In.
A Decametre is	=	0	0	10	2	9.7
A Hecatometre	=	0	0	109	1	.1
A Chiliometre	=	0	4	213	1	10.2
A Myriometre	=	6	1	156	0	.6

Eight Chiliometres are nearly 5 English miles.

MEASURES OF CAPACITY.

		English Cubic Inches.
Millilitre	=	.06102
Centilitre	=	.61024
Decilitre	=	6.10244
Litre	=	61.10244
Decalitre	=	610.24429
Hecatolitre	=	6102.44288
Chiliolitre	=	61024.42878
Myriolitre	=	610244.28778

A Litre is nearly 2⅛ Wine pints.

14 Decilitres are nearly 3 Wine pints.

A Chiliolitre is a tun, 12.75 Wine gallons.

WEIGHTS.

		English Grains.
Milligramme	=	.0154
Centigramme	=	.1544
Decigramme	=	1.5444
Gramme	=	15.4440
Decagramme	=	154.4402
Hecatogramme	=	1544.4023
Chiliogramme (Kilogram)	=	15444.0234
Myriogramme	=	154440.2344

A Decagramme is 6 dwts. 10.44 gr. tr.; or 5.65 dr. avoir.

A Hecatogramme is 3 oz. 8.5 dr. avoir.

A Chiliogramme is 2 libs. 3 oz. 5 dr. avoir.

A Myriogramme is 22 — 1.15 oz. avoir.

100 Myriogrammes are 1 Ton, wanting 32.8 libs.

AGRARIAN MEASURES.

Are, 1 square Decametre . = 3.95 Perches.

Hectare . . . . = 2 Acres, 1 Rood, 30.1 Perches.

FIR WOOD.

Decistre, 1-10th Stere . = 3.5315 cub. ft. Eng.

Stere, 1 Cubic Metre . = 35.3150 cub. ft.

MENSURATION.

Areas or Surfaces.

PROBLEM I.

To find the area of any Parallelogram, whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid. See Plate I. Fig. 1, 2, 3, 4.

RULE. Multiply the length by the perpendicular breadth or height, and the product will be the area.

EXAMPLE.

What is the area of a Rhomboid, the length A B being 7, the perpendicular B C being 4?

$$7 \times 4 = 28 \text{ area of Rhomboid.}$$

PROBLEM II.

To find the area of a Triangle.

RULE. Multiply the base by the perpendicular height, and take half the product for the area.

EXAMPLE.

What is the area of a Triangle, the base being 9, and the perpendicular 8½?

$$9 \times 8\tfrac{1}{2} = 76\tfrac{1}{2}\ \frac{76\frac{1}{2}}{2} = 38\tfrac{1}{4} \text{ area.}$$

PROBLEM III.

To find the area of a Trapezoid.—See Plate I. Fig. 5.

Rule. Add together the two parallel sides; then multiply their sum by the perpendicular breadth, or the distance between them, and take half the product for the area.

EXAMPLE.

What is the area of a Trapezoid?

$\left.\begin{array}{l} AB = 9 \\ CD = 8 \end{array}\right\}$ parallel sides.
BD = 5 perpendicular.

$\overline{9+8} \times 5 = 85$
and $\frac{85}{2} = 42\frac{1}{2}$ area.

PROBLEM IV.

To find the area of any Trapezium.—See Fig. 6.

Rule. Divide the Trapezium into two triangles by a diagonal; then find the areas of the triangles by Prob. 2. and add them together for the area of the Trapezium.

EXAMPLE.

What is the area of a Trapezium?

AC = 8 base of both triangles.
$\left.\begin{array}{l} DF = 4 \\ BE = 5 \end{array}\right\}$ perpendiculars.

$\overline{5+4} \times 8 = 72$
and $\frac{72}{2} = 36$ area.

PROBLEM V.

To find the area of an Irregular Polygon.—See Fig. 7.

Rule. Draw Diagonals dividing the proposed Polygon into Trapeziums and Triangles; then find the areas of all these separately, and add them together for the contents of the whole Polygon.

EXAMPLE.

What is the area of an irregular Polygon.

$$\left.\begin{array}{l} AC = 9 \\ AD = 8 \end{array}\right\}\text{ bases.}$$

$$\left.\begin{array}{l} BF = 2 \\ DH = 4 \\ EG = 3 \end{array}\right\}\text{ perpendiculars.}$$

$$\begin{array}{r} \overline{4+2}\times 9=54 \\ 3\times 8=\underline{24} \\ \frac{78}{2}=39 \text{ area.} \end{array}$$

PROBLEM VI.

To find the area of a Regular Polygon.

Rule 1. Multiply the perimeter of the Polygon, or sum of its sides, by the perpendicular drawn from its centre on one of its sides, and take half the product for the area.

Rule 2. Square the side of the Polygon; then multiply that square by the tabular area set against its name in the following Table, and the product will be the area.

EXAMPLE.

What is the area of a regular Nonagon, its side being 5, and perpendicular 6.8686935?

By Rule 1.

$$45 \text{ perimeter} = 5 \times 9$$

$$45 \times 6.8686935 = 309.0912097$$

$$\text{now } \frac{309.09 \text{ \&c.}}{2} = 154.54 \text{ \&c. area.}$$

By Rule 2.

$$5^2 \times 6.1818242 = 154.5456 \text{ \&c. area.}$$

No. of sides.	NAMES.	AREAS.
3	Trigon, or Triangle .	0.4330127
4	Tetragon, or Square .	1.0000000
5	Pentagon	1.7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
10	Decagon	7.6942088
11	Undecagon . . .	9.3656399
12	Dodecagon	11.1961524

PROBLEM VII.

To find the diameter and circumference of any Circle, the one from the other.—See Fig. 8.

This may be done by either of the three following proportions, viz. As 7 is to 22, so is the diameter to the circumference; or, As 1 is to 3.1416, so is the diameter to the circumference; or, As 113 is to 355, so is the diameter to the circumference.

EXAMPLE.

What is the circumference of a Circle, its diameter being 6?

First, 7 : 22 : : 6 : 18.857 circumference.
Second, 1 : 3.1416 : : 6 : 18.8496 do.
Third, 113 : 355 : : 6 : 18.8495 do.

This last (113 : 355) is the most correct proportion to find the circumference from the diameter, or the diameter from the circumference.

PROBLEM VIII.

To find the length of any arc of a Circle.—See Fig. 9.

Rule. Multiply the decimal .01745 by the degrees in the given arc, and the product by the radius of the circle, for the length of the arc.

EXAMPLE.

What is the length of the arc of a Circle, the number of degrees being 25, and radius 7?

$.01745 \times 25 \times 7 = 3.05375$, length of arc.

Note. .01745 is found by dividing the circumference by 360° when the radius is 1 —— *i. e.*

$$\frac{6.2831854}{360} = .01745$$

PROBLEM IX.

To find the area of a Circle.

Rule 1. Multiply half the circumference by half the diameter, and the product is the area.

Rule 2. Square the diameter, and multiply that square by the decimal .7854 for the area.

Rule 3. Square the circumference, and multiply that square by the decimal .07958.

EXAMPLE.

What is the area of a Circle, the diameter being 9, the circumference = 28.27, half of which is 14.135?

By Rule 1.
$14.135 \times 4.5 = 63.6075$

By Rule 2.
$9^2 \times .7854 = 63.6174$

By Rule 3.
$28.27^2 \times 07958 = 63.599$, &c.

Note. The first Rule of this Problem is found by the theorem of the triangle: for, suppose the circle to be a regular Polygon of an indefinite number of sides, then, the sum of the sides will be the perimeter of the circle; consequently, the radius of the circle will be the altitude, and the perimeter the base of the triangle, the area of which is found by $A \times \frac{1}{2} B$, or $\frac{1}{2} A \times B$, (A being the altitude and B the base,) therefore the area of the circle will be $\frac{1}{2}$ cir. × $\frac{1}{2}$ diameter, or $\frac{1}{2}$ cir. × radius.

The second Rule is deduced from the first and Prob. 7.: the first Rule is $\frac{D\,C}{4}$ = area (D C being the diameter and circumference) Prob. 7. is $3.1416\,D = C$, therefore the area is $\frac{3.1416\,D^2}{4}$ or $= .7854\,D^2$

The third Rule is found thus: $D = \frac{C}{3.1416}$ and $A = \frac{D\,C}{4}$, therefore the area will be $\frac{C^2}{3.1416 \times 4}$ or $\frac{C^2}{12.5664}$ now the reciprocal of 12.5664 is .07958, or $\frac{A}{C^2} = .07958$; hence the rule $C^2 \times .07958$ = area.

PROBLEM X.

To find the area of a circular Ring, or of the space included between the circumferences of two circles; the one being contained within the other.

Rule. Take the difference between the areas of the two circles for the area of the Ring.

EXAMPLE.

What is the area of a Ring, the inside diameter being 5, and outside 7?

$$\begin{array}{r} .7854 \times 7^2 = 38.4846 \\ .7854 \times 5^2 = \underline{19.6350} \\ 18.8496 \end{array}$$

= area of ring, or $\frac{7+5}{2} = 6$ medium diam^r., then $3.1416 \times 6 \times \frac{7-5}{2} = 18.84$ area, or $7^2 - 5^2 = 24$, then $7854 \times 24 = 18.84$ area.

PROBLEM XI.

To find the area of a Sector of a Circle.—See Fig. 10.

Rule 1. Multiply the radius, or half the diameter, by half the arc of the sector, for the area; or multiply the whole diameter by the whole arc of the sector, and take $\frac{1}{4}$ of the product.

Rule 2. Compute the area of the whole circle: then say, as 360° is to the degrees in the arc of the sector, so is the area of the whole circle, to the area of the sector.

EXAMPLE.

What is the area of the sector of a Circle, the the radius A C being 5, and arc A B, 8?

Rule 1. $4 \times 5 = 20$ area, or $\frac{8 \times 10}{4} = 20$ area.

Rule 2. $78.5400 =$ area of circle, then $360° : 92° 18' :: 78.54 : 20$ area, (92° 18′ is the portion of the circle contained in arc 8.)

PROBLEM XII.

To find the area of a Segment of a Circle.—See Fig. 11.

Rule 1. Find the area of the sector, having the same arc with the segment, by the 2nd rule of last Problem. Find also the area of the triangle, formed by the chord of the segment and the two radii of the sector; then add these together for the answer, when the segment is greater than a semicircle; or subtract them, when it is less than a semicircle: As is evident,

RULE 2. Divide the height of the segment by the diameter, and find the quotient in the column of heights in the following Table. Take out the corresponding area in the next column on the right hand; and multiply it by the square of the circle's diameter for the area of the segment.

When the quotient is not found exactly in the Table, proportion may be made between the next less and greater area, in the same manner as is done with any other Table.

EXAMPLE.

What is the area of the segment of a Circle, the radius A E being 10, chord A B C, 12, and arc A D C 73° 74′?

Rule 1. $.7854 \times 400 = 314.16$ area of whole circle, and $360° : 73° 74' :: 314.16 : 6434054$ area of sector. Half the chord is 6, and $\sqrt{10^2 - 6^2} = 8. =$ B E the height of the triangle.

Rule 2. $10 - 8 = 2$ height of the segment. $\frac{2}{20} = .1$ the quotient; and opposite to it, in the right column of the following Table, is .04088, which multiplied by the square of the diameter, is $.04088 \times 20^2 = 16.352$ area, nearly same area as found by Rule 1.

Table of the area of circular Segments.

Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.
.01	.00133	.11	.04701	.21	.11990	.31	.20738	.41	.30319
.02	.00375	.12	.05339	.22	.12811	.32	.21667	.42	.31304
.03	.00687	.13	.06000	.23	.13646	.33	.22603	.43	.32293
.04	.01054	.14	.06683	.24	.14494	.34	.23547	.44	.33284
.05	.01468	.15	.07387	.25	.15354	.35	.24498	.45	.34278
.06	.01924	.16	.08111	.26	.16226	.36	.25455	.46	.35274
.07	.02417	.17	.08853	.27	.17109	.37	.26418	.47	.36272
.08	.02944	.18	.09613	.28	.18002	.38	.27386	.48	.37270
.09	.03502	.19	.10390	.29	.18905	.39	.28359	.49	.38270
.10	.04088	.20	.11182	.30	.19817	.40	.29337	.50	.39270

PROBLEM XIII.

To measure long irregular Figures.

RULE. Take or measure the breadth at both ends, and at several places, at equal distances; then add together all these intermediate breadths, and half the two extremes; which sum multiply by the length, and divide by the number of parts for the area. If the perpendiculars or breadths be not at equal distances, compute all the parts separately, as so many trapezoids, and add them all together for the whole area.

EXAMPLE.

What is the area of an irregular figure, the breadths at equal distances being 8.2, 7.4, 9.2, 10.2, 8.6, and the whole length 39?

$$\begin{array}{l} 8.2 \\ \underline{8.6} \\ 2\overline{)16.8} \text{ sum of the extremes.} \\ \overline{8.4} \text{ mean of the extremes.} \end{array}$$

$8.4+7.4+9.2+10.2=35.2$ sum of the breadths, now $35.2\times39=1372.8$ which divided by the number of parts $=\frac{1372.8}{4}=$ 343.2 area.

PROBLEM XIV.

To find the area of an Ellipsis or Oval.—See Fig. 12.

Rule. Multiply the longest diameter by the shortest; then multiply the product by the decimal .7854, for the area.

EXAMPLE.

What is the area of an Oval, its diameters being 7 and 5?

$$7\times5\times.7854=27.4890 \text{ area.}$$

PROBLEM XV.

To find the area of an Elliptic Segment.

Rule 1. Find the area of a corresponding circular segment, having the same height, and the same vertical axis or diameter; then say, as the said vertical axis is to the other axis, parallel to the segment's base; so is the area of the circular segment before found, to the area of the elliptic segment sought.

Rule 2. Divide the height of the segment by the vertical axis of the ellipse, and find in the Table of circular segments, Prob. 12, the circular segment having the above quotient for its versed sine; then multiply altogether, this segment and the two axes of the ellipse.

EXAMPLE.

What is the area of an elliptic segment, its height being 2, vertical axis 20, and parallel axis 5?

Rule 1. $\begin{array}{r} 20)\underline{2} \\ .1 \end{array}$ gives $.04088 \times 20^2 = 16.352$ area of circular segment: 20 : 5 : : 16.352 : 4.0880 area of elliptic segment.

Rule 2. $\begin{array}{r} 20)\underline{2} \\ .1 \end{array}$ gives $.04088 \times \overline{5 \times 20} = 4.0880$ area.

PROBLEM XVI.

To find the area of a Parabola, or its Segment.
See. Fig. 13.

Rule. Multiply the base by the perpendicular height; then take two-thirds of the product for the area.

EXAMPLE.

What is the area of a Parabola, its base being 6, and height 9?

$$6 \times 9 = \frac{54 \times 2}{3} = 36 \text{ area of segment.}$$

SOLIDS.

By Mensuration of Solids, are determined the spaces included by contiguous surfaces; and the sum of the measures of these including surfaces, is the whole surface or superficies of the body.

The measure of a body is called its solidity, capacity, or content.

Solids are measured by cubes, whose sides are inches, feet, or yards, &c.; and hence the solidity of a body is said to be so many cubic inches, feet, yards, &c. as will fill its capacity or space, or another of an equal magnitude.

The least solid measure is the cubic inch; other cubes being taken from it, according to the proportion in the following Table, which is formed by cubing the linear proportions.

Table of Cubes or Solids.

1728	Cubic Inches	make	1	Cubic Foot.
27	Cubic Feet	——	1	Cubic Yard.
166⅜	Cubic Yards	——	1	Cubic Pole.
64000	Cubic Poles	——	1	Cubic Furlong.
512	Cubic Furlongs	——	1	Cubic Mile.

PROBLEM I.

To find the superficies of a Prism or Cylinder.—See Fig. 14 and 15.

Rule. Multiply the perimeter of one end of the Prism, by the length of the solid, and the product will be the surface of all its sides. To which add also the area of the two ends of the Prism, when required.

Or, compute the areas of all the sides and ends separately, and add them all together.

EXAMPLE.

What is the superficies of an equilateral triangular Prism, its length being 9, and side 3?

$3 \times 3 = 9$ perimeter, then $9 \times 9 = 81$ superficies, or $3 \times 9 = 27$ area of one side, and $27 \times 3 = 81$ superficies or areas of the 3 sides.

PROBLEM II.

To find the surface of a Pyramid or Cone.—See Fig. 16 and 17.

Rule. Multiply the perimeter of the base by the slant height, or length of the side, and half the product will be the surface of the sides, or the sum of the areas of all the triangles which form it. To which add the area of the end or base, if required.

EXAMPLE.

What is the surface of a Pyramid, its slant height being 20, and the perimeter of its base 15?

$$\frac{15 \times 20}{2} = 150 \text{ surface of pyramid.}$$

PROBLEM III.

To find the surface of the Frustum of a Pyramid or Cone, being the lower part, when the top is cut off by a plane parallel to the base.

Rule. Add together the perimeters of the two ends, and multiply their sum by the slant height, taking half the product for the answer—As is evident, because the sides of the solid are trapezoids, having the opposite sides parallel.

EXAMPLE.

What is the surface of the frustum of a Cone, the slant height being 12, the diameters 8 and 6?

$3.1416 \times 8 = 25.12$
$3.1416 \times 6 = \underline{18.85}$
43.97 perimeters of both ends,

$43.97 \times 12 = 527.64$, which halved, is 263.82, the surface of frustum.

PROBLEM IV.

To find the solid content of any Prism or Cylinder.

Find the area of the base, or end, whatever the figure of it may be, and multiply it by the length of the Prism or Cylinder, for the solid content.

EXAMPLE.

What is the solid content of a Cylinder, its diameter being 3, and length 7?

$.7854 \times 9 = 7.0686$ area, $7.0686 \times 7 = 49.4802$ solidity.

PROBLEM V.

To find the content of any Pyramid or Cone.—See Fig. 16 and 17.

Rule. Find the area of the base, and multiply that area by the perpendicular height; then take one-third of the product for the content.

EXAMPLE.

What is the content of a Cone, the area of its base being 9, and vertical height 17?

$$17 \times 9 = 153, \frac{153}{3} = 51 \text{ solid content.}$$

PROBLEM VI.

To find the solidity of the Frustum of a Cone or Pyramid.

Rule. Add into one sum the areas of the two ends, and the mean proportional between them; and take one-third of that sum for a mean area; which being multiplied by the perpendicular height or length of the frustum, will give its content.

EXAMPLE.

What is the solidity of the frustum of a Cone, the vertical height 19, the areas of its ends being 12 and 9?

12
9
10.5 mean proportional.
3) 31.5
10.5 × 19 = 199.5 solid content.

PROBLEM VII.

To find the surface of a Sphere or any Segment. See Fig. 18.

Rule 1. Multiply the circumference of the Sphere by its diameter, and the product will be the whole surface of it.

Rule 2. Square the diameter, and multiply that square by 3.1416, for the surface.

Rule 3. Square the circumference; then either multiply that square by the decimal .3183, or divide it by 3.1416, for the surface.

Note. For the surface of a Segment or Frustum, multiply the whole circumference of the Sphere by the height of the part required.

EXAMPLE.

What is the superficial content of a Sphere, its diameter being 7?

$$\textit{Rule } 1. \left\{\begin{array}{ll}\text{Circum.} & 22\\ \text{Diam}^{\text{r.}} & 7\end{array}\right. \quad 22 \times 7 = 154 \text{ superf. cont.}$$

$$\textit{Rule } 2. \quad 7^2 \times 3.1416 = 153.9384 \qquad \text{do.}$$

$$\textit{Rule } 3. \left\{\begin{array}{ll} 22^2 \times .3183 = 154.0572 & \text{do.}\\ \text{or } \dfrac{22^2}{3.1416} = 154.06 & \text{do.}\end{array}\right.$$

PROBLEM VIII.

To find the solidity of a Sphere or Globe.

Rule 1. Multiply the surface by the diameter, and take 1-6th of the product for the content; or, which is the same thing, multiply the square of the diameter by the circumference, and take 1-6th of the product.

Rule 2. Take the cube of the diameter, and multiply it by the decimal .5236, for the content.

Rule 3. Cube the circumference, and multiply it by .01688, for the content.

EXAMPLE.

What is the solid content of a Globe 7 inches diameter?

This is the same diameter of the Sphere as in last Example; therefore the surface will be 154 inches.

Rule 1. $\frac{154 \times 7}{6} = 179\frac{1}{6}$ or $\frac{7^2 \times 22}{6} = 179\frac{1}{6}$ cub. cont.

Rule 2. $7^3 = 343 \times .5236 = 179.5948$ cub. content.

Rule 3. $22^3 = 10648 \times .01688 = 179.73824$ do.

PROBLEM IX.

To find the solid content of a Spherical Segment.
See Fig. 18.

Rule 1. From 3 times the diameter of the Sphere, take double the height of the Segment; then multiply the remainder by the square of the height, and the product by the decimal .5236, for the content.

Rule 2. To 3 times the square of the radius of the Segment's base, add the square of its height;

then multiply the sum by the height, and the product by .5236, for the content.

EXAMPLE.

What is the solid content of a Spherical Segment 2 feet high, taken from a Sphere 8 feet diameter?

Rule 1. $8\times3=24-2\times2=20\times2^2=80\times.5236=$ 41.888, radius of sphere is 4, now $4^2=16$, and from the radius take the height of the segment, $4-2=2$, which $2^2=4$, therefore $16-4=12=$ square of the radius of Segment's base.

Rule 2. $\overline{12\times3+2^2}\times2\times.5236=41.888$ content.

In arranging the first Edition, it was thought superfluous to give Examples for the elucidation of the preceding very simple Rules; but after publication, it was found that the want of Examples operated against the general usefulness of the book; for this reason they are now annexed.

SPECIFIC GRAVITY.

THE specific gravity of a body is the proportional weight between that body and another of a known density; and water is admirably adapted to be the standard, as a solid foot of it weighs 1000 ounces avoirdupois.

TO FIND THE SPECIFIC GRAVITY OF A BODY.

PROBLEM I.

When the body is heavier than water.

RULE. Weigh it both in and out of water, and take the difference, which will be the weight lost in water; then say,

As the weight lost in water,
Is to the whole or absolute weight;
So is the specific gravity of water,
To the specific gravity of the body.

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

Rule. Take the three differences of every pair of the three specific gravities, namely, the specific gravities of the compound and each ingredient; and multiply each specific gravity by the difference of the other two; then say,

As the greatest product,
Is to the whole weight of the compound;
So is each of the other two products,
To the weights of the two ingredients.

EXAMPLE.

A composition of 112 lbs. being made of tin and copper, whose specific gravity is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and copper 9000.

8784, ... Composition,
9000, ... Copper,
7320, ... Tin.

9000 — 7320 = 1680 × 8784 = 14757120
8784 — 7320 = 1464 × 9000 = 13176000
9000 — 8784 = 214 × 7320 = 1566480

As 14757120 : 112 :: 13176000 : 100 = Copper } Weight of
112 — 100 = 12 = Tin } Ingredients

A Table of Specific Gravities of Bodies.

Platina (pure)	23000	Tin	7320
Fine Gold	19400	Clear Crystal Glass	3150
Standard Gold	17724	Granite	3000
Quicksilver (pure)	14000	Marble and Hard Stone	2700
Quicksilver (common)	13600	Common Green Glass	2600
Lead	11325	Flint	2570
Fine Silver	11091	Common Stone	2520
Standard Silver	10535	Clay	2160
Copper	9000	Brick	2000
Copper Halfpence	8915	Common Earth	1984
Gun Metal	8784	Nitre	1900
Cast Brass	8000	Ivory	1825
Steel	7850	Brimstone	1810
Iron	7645	Solid Gunpowder	1745
Cast Iron	7425	Sand	1520
Coal	1250	Ash	800
Boxwood	1030	Maple	755
Sea Water	1030	Elm	600
Common Water	1000	Fir	550
Oak	925	Charcoal	400
Gunpowder close shaken	937	Cork	240
Do. in a loose heap	836	Air at a mean state	1$\frac{2}{9}$

Note. The several sorts of wood are supposed to be dry. Also, as a cubic foot of water weighs just 1000 ounces avoirdupois, the numbers in this Table express, not only the specific gravities of the several bodies, but also the weight of a cubic foot of each in avoirdupois ounces; and therefore, by proportion, the weight of any other quantity, or the quantity of any other weight, may be known. Also, 100 cubic inches of common air weigh nearly 31½ grains troy, or 1$\frac{1}{7}$ drams avoirdupois. HUTTON.

TABLES OF THE WEIGHT OF MALLEABLE AND CAST IRON PLATES, BARS, &c.

Table *of the Weight of a Square Foot of Cast and Malleable Iron, Copper and Lead, from* 1-16*th, to* 1 *Inch thick.*

Thick.	Cast Iron.		Mall. Iron.		Copper.		Lead.	
	Libs.	*Oz.*	*Libs.*	*Oz.*	*Libs.*	*Oz.*	*Libs.*	*Oz.*
1 Sixteenth	2	6.6	2	7.8	2	15	3	11
2 —	4	13.3	4	15.6	5	14	7	6
3 —	7	4.	7	7.4	8	13	11	1
4 —	9	10.6	9	15.2	11	12	14	12
5 —	12	1.3	12	7.1	14	11	18	7
6 —	14	8.	14	14.9	17	10	22	2
7 —	16	14.7	17	6.7	20	9	25	13
8 —	19	5.3	19	14.5	23	8	29	8
9 —	21	12.	22	6.3	26	7	33	3
10 —	24	2.7	24	14.2	29	6	36	14
11 —	26	9.3	27	6.	32	5	40	9
12 —	29	-	29	13.8	35	4	44	4
13 —	31	6.7	32	5.6	38	3	47	15
14 —	33	13.4	34	13.4	41	2	51	10
15 —	36	4.	37	5.3	44	1	55	5
1 Inch	38	10.7	39	13.1	47	-	59	-

Table *of the Weight of a Lineal Foot of Malleable and Cast Iron Bars, from* 6-16*ths to* 3 *Inches square.*

Sixteenths on the side.	Area in Square Sixteenths.	MALL. IRON. *Ounces Weight.*	CAST IRON. *Ounces Weight.*	ROUND RODS. The 16ths on the side is the diameter of Rod. *Ounces Weight.*
6	36	7.4736	. . . . .	5.83
7	49	10.1724	. . .	7.99
8	64	13.2864	12.8960	10.43
9	81	16.8156	. . . . .	13.20
10	100	20.7600	. . .	16.30
11	121	25.1196	. . . . .	19.72
12	144	29.8944	29.0160	23.47
13	169	35.0844	. . . . .	27.53
14	196	40.6896	. . .	31.94
15	225	46.7100	. . . . .	36.44
1 Inch	256	53.1456	51.5840	41.50
1	289	59.9964	. . . . .	46.80
2	324	67.2624	. . .	52.47
3	361	74.9436	. . . . .	58.46
4	400	83.0400	80.6000	64.81
5	441	91.5516	. . . . .	71.41
6	484	100.4784	. . .	78.37
7	529	109.8204	. . . . .	85.66
8	576	119.5774	116.0640	93.27
9	625	129.7500	. . . . .	101.21
10	676	140.3376	. . .	109.46
11	729	151.3404	. . . . .	118.05
12	784	162.7584	157.9760	126.95
13	841	174.5916	. . . . .	136.19
14	900	186.8400	. . .	145.74
15	961	199.5036	. . . . .	155.62
2 Inches	1024	212.5824	206.3360	165.82
1	1089	226.0764	. . . . .	176.34
2	1156	239.9856	. . .	187.19
3	1225	254.3100	. . . . .	198.36
4	1296	269.0496	261.1440	209.86
5	1369	284.2044	. . . . .	221.68
6	1444	299.7744	. . .	233.83
7	1521	315.7596	. . . . .	246.30
8	1600	332.1600	322.4000	259.09
9	1681	348.9756	. . . . .	272.20
10	1764	366.2064	. . .	285.64
11	1849	383.8524	. . . . .	299.41
12	1936	401.9136	390.1040	313.49
13	2025	420.3900	. . . . .	327.91
14	2116	439.2816	. . .	342.64
15	2209	458.5884	. . . . .	357.70
3 Inches	2304	478.3104	464.2560	373.09

The foregoing Tables have been calculated from Hutton's Specific Gravities; those of Cast and Malleable Iron and Lead agree very nearly with those given by other authors; but the specific gravity of Copper, though heavier than that given by Hatchett, which is 8.800; still, from Copper being frequently alloyed with Lead, it is supposed that Hutton's, which is 9000, will be nearest the weight of Copper commonly used.

FALLING BODIES.

The motion described by Bodies freely descending by their own gravity, is, viz.—The Velocities are as the Times, and the Spaces as the Squares of the Times.—Therefore if the Times be as the

numbers	1	2	3	4	&c.
The Velocities will be also as . .	1	2	3	4	&c.
The Spaces as their Squares .	1	4	9	16	&c.
and the Spaces for each time, as .	1	3	5	7	&c.

namely, as the series of the odd numbers, which are the differences of the squares, denoting the whole spaces:—So that if the first series of numbers

be seconds of time: *i. e.* . . .	$1''$	$2''$	$3''$	&c.
Velocities in feet will be . .	$32\frac{1}{6}$	$64\frac{1}{3}$	$96\frac{1}{2}$	&c.
Spaces in the whole times will be	$16\frac{1}{12}$	$64\frac{1}{3}$	$144\frac{3}{4}$	&c.
Spaces for each second will be	$16\frac{1}{12}$	$48\frac{1}{4}$	$80\frac{5}{12}$	&c.

HUTTON.

The following Table *shows the Spaces fallen through, and the Velocities acquired, at the end of each of* 30 *Seconds.*

Time in Seconds.	SPACE.				VELOCITY.		
	Each Time.	As the Squares of the Time.	Fallen through in Feet & Inches.		As the Times.	Acquired in Feet & Inches.	
1	1	1	16	1	1	32	2
2	3	4	64	4	2	64	4
3	5	9	144	9	3	96	6
4	7	16	257	4	4	128	8
5	9	25	402	1	5	160	10
6	11	36	579	0	6	193	0
7	13	49	788	1	7	225	2
8	15	64	1029	4	8	257	4
9	17	81	1302	9	9	289	6
10	19	100	1608	4	10	321	8
11	21	121	1946	1	11	353	10
12	23	144	2316	0	12	386	0
13	25	169	2718	1	13	418	2
14	27	196	3152	4	14	450	4
15	29	225	3618	9	15	482	6
16	31	256	4117	4	16	514	8
17	33	289	4648	1	17	546	10
18	35	324	5211	0	18	579	0
19	37	361	5806	1	19	611	2
20	39	400	6433	4	20	643	4
21	41	441	7092	9	21	675	6
22	43	484	7784	4	22	707	8
23	45	529	8508	1	23	739	10
24	47	576	9264	0	24	772	0
25	49	625	10052	1	25	804	2
26	51	676	10872	4	26	836	4
27	53	729	11724	9	27	868	6
28	55	784	12609	4	28	900	8
29	57	841	13526	1	29	932	10
30	59	900	14475	0	30	965	0

EXAMPLE I.

To find the space descended by a body in 7″ and the velocity acquired.

$$\begin{array}{rrcrrl} & & & \text{ft.} & \text{in.} & \\ 16\ 1 & \times\ 49 & = & 788 & 1 & \text{of space.} \\ 32\ 2 & \times\ 7'' & = & 225 & 2 & \text{of velocity.} \end{array}$$

Look into the Table at 7″ and you have the answers.

EXAMPLE II.

To find the time of generating a velocity of 100 feet per second, and the whole space descended.

$$\frac{100 \quad \times\ 12}{32\ 2 \times\ 12} = 3''\tfrac{21}{193} \text{ Time.}$$

$$\frac{3''\frac{21}{193} \times\ 100}{2} = 155\tfrac{85}{193} \text{ Space descended.}$$

EXAMPLE III.

To find the time of descending 400 feet, and the velocity at the end of that time.

$$\sqrt{\frac{400 \quad \times\ 12}{16\ 1 \times\ 12}} = 4''\ 987 \text{ Time.}$$

$$\frac{400 \quad \times\ 2}{4''\ 987} = 169.662 \text{ Velocity.}$$

Or these answers can be found from the Table by Proportion.

PENDULUM.

The vibrations of Pendulums are as the square roots of their lengths; and as it has been found by many accurate experiments, that the pendulum vibrating seconds in the latitude of London, is $39\frac{1}{8}$ inches long nearly, the length of any other pendulum may be found by the following Rule, viz. As the number of vibrations given, is to 60, so is the square root of the length of the pendulum that vibrates seconds, to the square root of the length of the pendulum that will oscillate the given number of vibrations:—or, As the square root of the length of the pendulum given, is to the square root of the length of the pendulum that vibrates seconds, so is 60 to the number of vibrations of the given pendulum.

Since the pendulum that vibrates seconds, or 60, is $39\frac{1}{8}$ inches long, the calculation is rendered simple; for $\sqrt{39\frac{1}{8}} \times 60 = 375$, a constant number, therefore 375, divided by the square root of the pendulum's length, gives the vibrations per minute, and divided by the vibrations per minute, gives the square root of the length of the pendulum.

EXAMPLE I.

How many vibrations will a pendulum of 49 inches long make in a minute?

$$\frac{375}{\sqrt{49}} = 53\ \tfrac{4}{7} \text{ vibrations in a minute.}$$

EXAMPLE II.

What length of a pendulum will it require to make 90 vibrations in a minute?

$$\frac{375}{90} = 4.16 \quad \overline{4.16}^{2} = 17.3056 \text{ inches long.}$$

EXAMPLE III.

What is the length of a pendulum, whose vibrations will be the same number as the inches in its length?

$\sqrt[3]{\overline{375}^{2}} = 52$ inches long, and 52 vibrations.[*]

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[*] For $\sqrt{x} : \sqrt{\overline{39\frac{1}{8}}} :: 60 : x$ or $\sqrt{x} \times x = 375$

$$\sqrt{x} = \frac{375}{x} = x = \frac{\overline{375}^{2}}{x^{2}}$$

or $x \times x^{2} = \overline{375}^{2} = x^{3} = \overline{375}^{2}$

$$x = \sqrt[3]{\overline{375}^{2}} = 52.002$$

MECHANICAL POWERS, &c.

The Science of Mechanics is simply the application of Weight and Power, or Force and Resistance. The weight is the resistance to be overcome; the power is the force requisite to overcome that resistance. When the force is equal to the resistance, they are in a state of equilibrium, and no motion can take place; but when the force becomes greater than the resistance, they are not in a state of equilibrium, and motion takes place; consequently, the greater the force is to the resistance, the greater is the motion or velocity.

The Science of Equilibrium is called Statics; the Science of Motion is called Dynamics.

Mechanical Powers are the most simple of mechanical applications to increase force, and overcome resistance. They are usually accounted six in number, viz. The Lever,—The Wheel and Axle,—The Pulley,—The Inclined Plane,—The Wedge,—and the Screw.

$10 \times 2 = 20$

[illegible]

LEVER.

To make the principle easily understood, we must suppose the Lever an inflexible rod without weight; when this is done, the rule to find the equilibrium between the power and the weight, is,—Multiply the weight by its distance from the fulcrum, prop, or centre of motion, and the power by its distance from the same point: if the products are equal, the weight and power are in equilibrio; if not, they are to each other as their products.

EXAMPLE I.

A weight of 100 libs on one end of a lever, is 6 inches from the prop, and a weight of 20 libs at the other end, is 25 inches from the prop—What additional weight must be added to the 20 libs, to make it balance the 100 libs?

$\frac{100 \times 6}{25} = 24 - 20 = 4$ libs weight to be added.

EXAMPLE II.

A Block of 960 libs is to be lifted by a lever 30 feet long, and the power to be applied is 60 libs—on what part of the lever must the fulcrum be placed?

$\frac{960}{60} = 16$, that is, the weight is to the power as 16 is to 1; therefore the whole length $\frac{30}{16+1} = 1\frac{13}{17}$, the distance from the block, and $30 - 1\frac{13}{17} = 28\frac{4}{17}$, the distance from the power.

EXAMPLE III.

A Beam 32 feet long, and supported at both ends, bears a weight of 6 tons, 12 feet from one end,—What proportion of weight does each of the supports bear?

$\frac{12 \times 6}{32} = 2\frac{1}{4}$ tons, support at end farthest from the weight.

$\frac{20 \times 6}{32} = 3\frac{3}{4}$ tons, support at end nearest the weight.

EXAMPLE IV.

A Beam supported at both ends, and 16 feet long, carries a weight of 6 tons, 3 feet from one end, and another weight of 4 tons, 2 feet from the other end: What proportion of weight does each of the supports bear?

$\frac{3 \times 6}{16} + \frac{14 \times 4}{16} = \frac{74}{16} = 4\frac{10}{16}$ tons, end at the 4 tons.

$\frac{2 \times 4}{16} + \frac{13 \times 6}{16} = \frac{86}{16} = 5\frac{6}{16}$ tons, end at the 6 tons.

When the weight of the lever is taken to account, *see Centre of Gravity.*

WHEEL AND AXLE.

See Fig. 1. *Plate* 2.

The power gained by the Wheel and Axle, Wheel and Pinion, or Crane, is the effect of a double lever; for suppose the end of the lever *a b* is the radius of the rope barrel, and the radius of the wheel *b c;* again, the radius of the pinion working into the wheel, is *d e*, and the length of the handle or winch is *e f*.

If the distance between *a b* is only one-third of the distance between *b c*, it is evident, that the point at *c* will go through three times the space to that of the point at *a*, when the lever revolves round its fulcrum *b:* the points *d* & *f*, in the other lever, are in the same proportion. The short end *d* acts upon the long end *c:* and if the end *f* goes through 9 inches, the end *d* will go through 3 inches, also the end *c*. If the end *c* goes through 3 inches, the end *a* will go through only 1 inch; therefore the power is to the weight as 9 is to 1; that is, If 9 libs be hung at the end of the arm *a*, and 1 lib hung at the end of the arm *f*, they will balance each other. From this it is evident, that if you gain power, you lose speed; and by gaining speed, you lose power: hence the Rule is deduced—Multiply the power applied by its velocity, and the weight to be raised by its velocity.

EXAMPLE I.

A Weight of 94 tons is to be raised 360 feet in 15 minutes, by a power, the velocity of which is 220 feet per minute:—What is the power required?

$\frac{360}{15} = 24$ feet per minute, velocity of weight.

$24 \times 94 = \frac{2256}{220} = 10.25\dot{4}\dot{5}$ tons power required.

EXAMPLE II.

A Stone weighing 986 libs, is required to be lifted: What power must be applied, when the power is to the weight as 9 is to 2?

$$\frac{986 \times 2}{9} = \frac{1972}{9} = 219\tfrac{1}{9} \text{ tons power.}$$

EXAMPLE III.

A Power of 18 libs is applied to the winch of a crane, the length of which is 8 inches; the pinion makes 12 revolutions for 1 of the wheel, and the barrel is 6 inches diameter.

$\frac{8 \times 2 \times 22}{7} = 50.28$ circumference of the winch's circle.

$50.28 \times 12 = 603.36$ inches velocity of power on winch to 1 revolution of the barrel.

$$603.36 \times 18 = \frac{10860.48}{\frac{6 \times 22}{7} = 18.857 \ldots 19.} = 571.604 \text{ libs weight,}$$

that can be raised by a power of 18 libs on this crane.

PULLEY.

There are two kinds of Pulleys, the *fixed* and the *moveable.* From the fixed pulley no power is derived; it is as a common beam used in weighing goods, having the two ends of equal weight, and at the same distance from the centre of motion: the only advantage gained by the fixed pulley, is in changing the direction of the power.

From the moveable pulley power is gained; it operates as a lever of the second order; for if one end of a string be fixed to an immoveable stud, and the other end to a moveable power, the string doubled and the ends parallel, the pulley that hangs between is a lever; the fixed end of the string being the fulcrum, and the other the moveable end of the lever: hence the power is double the distance from the fulcrum, than is the weight hung at the pulley; and therefore the power is to the weight as 2 is to 1. This is all the advantage gained by one moveable pulley; for two, twice the advantage; for three, thrice the advantage; and so on for every additional moveable pulley.

From this the following Rule is derived:—Divide the weight to be raised by twice the number of moveable pulleys, and the quotient is the power required to raise the weight.

EXAMPLE I.

What power is requisite to lift 100 libs, when two blocks of three pulleys, or sheives each, are applied, the one block moveable and the other fixed?

$$\frac{100}{6} = 16\tfrac{4}{6} \text{ libs, the power required,}$$

3 sheives × 2 = 6.

EXAMPLE II.

What weight will a power of 80 libs lift, when applied to a 4 and 5 sheived block and tackle, the 4 sheived block being moveable? *

80 × 8 = 640 libs weight raised.

INCLINED PLANE.

When a body is drawn up a vertical plane, the whole weight of the body is sustained by the power that draws or lifts it up: hence the power is equal to the weight.

---

* *See Fig.* 2. *Plate* 2.—If the strings be not parallel, but in the directions D A, D B, then the power A requisite to lift the weight C is as D E is to D C, and the strain upon the fixed point B is as C E is to C D.—HUTTON.

From this simple definition, it is easy to find the proportion between the power and the weight, when the strings are at any angle.

Screw, hence, the Rule is derived:—As the circumference of the Screw is to the Pitch, or distance between the threads; so is the Weight to the Power.

When the Screw turns, the cord or thread runs in a continued ascending line round the centre of the cylinder, and the greater the radius of the cylinder, the greater will be the length of the plane to its height, consequently the greater the power.—A lever fixed to the end of the screw will act as one of the second order, and the power gained will be as its length, to the radius of the cylinder; or the circumference of the circle described by it, to the circumference of the cylinder; hence, an addition to the Rule is produced, which is,—If a lever is used, the circumference of the lever is taken for, or instead of, the circumference of the screw.

EXAMPLE I.

What is the power requisite to raise a weight of 8000 libs by a Screw of 12 inches circumference and 1 inch pitch?

As 12 : 1 : : 8000 : $666\frac{8}{12}$ libs = power at the circumference of the screw.

EXAMPLE II.

How much would be the power if a lever of 30 inches was applied to the screw?

Circumference of 30 inches = $188\frac{4}{7}$

As $188\frac{4}{7}$: 1 : : 8000 : $42\frac{560}{1320}$ libs = power with a lever of 30 inches long.

STRENGTH OF MATERIALS.

AFTER considering the Mechanical Powers, which are the analytic parts of all Machines, the next step is to consider the Strength of the Materials of which Machinery is composed—this knowledge being of the greatest importance to the Mechanic, by enabling him to adjust, with respect to magnitude, the various parts of the machine, that the strength of each part may be proportional to the stress it has to sustain.

COHESIVE STRENGTH OF BEAMS, BARS, &c.

The cohesive strength of a body, is that force by which its fibres or particles resist separation, therefore the more particles that are in a body, the greater will be the power requisite to tear them asunder; or, according to the Rule, that the strength of bodies are as the area of their cross sections.

The knowledge in this property of bodies is very limited, there being very few experiments made on which to build a data, and these few do not agree.

The following are the results of experiments made by Mr. Emerson, which state the load that may be safely borne by a square inch rod of each.

	Pounds Avoirdupois.
Iron rod, an inch square, will bear	76,400
Brass	35,600
Hempen Rope	19,600
Ivory	15,700
Oak, Box, Yew, Plum-tree, . .	7,850
Elm, Ash, Beech	6,070
Walnut, Plum	5,360
Red Fir, Holly, Elder, Plane, Crab	5,000
Cherry, Hazel	4,760
Alder, Asp, Birch, Willow, . . .	4,290
Lead	430
Free Stone	914

Mr. Barlow's opinion of this table is, "We shall only observe here, that they all fall very short of the ultimate strength of the woods to which they refer." *See Barlow's Essay on the strength of Timber. Art.* 3.

Mr. Emerson also gives the following practical Rule, viz. "That a Cylinder, whose diameter is *d* inches, loaded to one-fourth of its absolute strength, will carry as follows:

	Cwt.
Iron, . . .	$135 \times d^2$
Good Rope, .	$22 \times d^2$
Oak, . . .	$14 \times d^2$
Fir, ...	$9 \times d^2$

Captain S. Brown made an experiment on Welsh Pig Iron, and the result is described as follows:

"A Bar of Cast Iron, Welsh Pig, 1¼ inch square, 3 feet 6 inches long, required a strain of 11 tons, 7 cwt. (25,424 libs,) to tear it asunder, broke exactly transverse, without being reduced in any part; quite cold when broken, particles fine, dark bluish grey colour."—From this experiment, it appears that 16,265 libs. will tear asunder a square inch of Cast Iron.

Mr. G. Rennie also made some experiments on Cast Iron, and the result was, "that a Bar one inch square, cast horizontal, will support a weight of 18,656 libs—and one cast vertical, will support a weight of 19,488 libs."

There have been several experiments made on Malleable Iron, of various qualities, by different Engineers.

The mean of Mr. Telford's experiments, is 29¼ tons.
The mean of Capt. S. Brown's do. is 25 do.
and the mean between these two means, is 27 tons, nearly; which may be assumed as the medium strength of a Malleable Iron Bar 1 inch square. *See Barlow's Essay, page 235.*

From a mean, derived by experiments, performed by Mr. Barlow, it appears that the strength of direct cohesion, on a square inch of

		Libs.
Box	is about	20,000
Ash	—	17,000
Teak	—	15,000
Fir	—	12,000
Beech	—	11,500
Oak	—	10,000
Pear	—	9,800
Mahogany	—	8,000

Each of these weights may be taken as a correct data for the cohesive strength of the wood to which they belong; but this is the absolute and ultimate strength of the fibres; and therefore, if the quantity that may be safely borne be required, not more than two-thirds of the above values must be used.

TRANSVERSE STRENGTH OF BEAMS, BARS, &c.

If a beam be supported at both ends, and loaded in the middle, it will bend; (which is called deflection;) and if the load be increased, it will break, (which is called fracture.)—If a beam 2 inches deep and one inch broad, support a given weight, another beam of the same depth, and double the breadth, will support double the weight:—hence, beams of the same depth are to each other as their breadths:—again, If a beam 2 inches deep, and 1 inch broad, support a given weight, another beam of 4 inches deep, and 1 inch broad, will support four times the weight;—hence, beams of equal breadths are to each other as the squares of their depths:—again, If a beam of a given cross section 1 foot long, support a known weight, another beam of the same cross section but 2 feet long, will support only half the known weight:—hence, beams of equal dimensions are to each other inversely as their lengths; therefore, the strength of beams is directly as their breadths and square of their depths, and inversely as their lengths; and if cylindrical, as the cubes of their diameters.

PRACTICAL PROBLEMS *for the Transverse Strength of Timber.*[*]

TABLE OF MULTIPLICANDS.

English Oak	1426
Canadian do.	1766
Ash	2026
Beech	1556
Elm	1013
Pitch Pine	1632
Red Pine	1341
Fir	1100
Larch	1127

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[*] See Barlow's Essay on the strength and stress of Timber, *Art.* 149.

PROBLEM I.

To find the ultimate transverse strength of any Rectangular Beam of Timber, fixed at one end, and loaded at the other.

Rule. Multiply the number in the Table of Multiplicands, by the breadth and square of the depth, both in inches, and divide that product by the length also in inches; the quotient will be the weight in libs.*

EXAMPLE I.

What weight will it require to break a beam of Fir, the breadth being 2 inches, depth 6 inches, and length 20 feet?

$$\frac{1100 \times 36 \times 2}{240} = 330 \text{ libs.}$$

EXAMPLE II.

What is the weight requisite to break a beam of Ash, 7 inches square, 3 feet from the wall?

$$\frac{2026 \times 7^3}{36} = 19303\tfrac{10}{36} \text{ libs.}$$

---

* When the beam is loaded uniformly throughout its length, the same rule will still apply, only the result must be doubled.

PROBLEM II.

To compute the ultimate transverse strength of any Rectangular Beam, when supported at both ends and loaded in the centre.

Rule. Multiply the number in the Table of Multiplicands, by the square of the depth in inches, and four times the breadth; divide that product by the length in inches, and the quotient will be the weight.

EXAMPLE I.

What weight will break a beam of English Oak 7 inches broad, 9 inches deep, and 30 feet between the props?

$$\frac{1426 \times 81 \times 28}{360} = 8983 \tfrac{288}{360} \text{ libs.}$$

EXAMPLE II.

A beam of Beech, 7 inches deep, 4 inches broad, and 10 feet long, supports a weight of 4 tons, what additional weight will require to be added to break the beam?

$$\frac{1556 \times 49 \times 16}{120} = 10332 - 8960 = 1372 \text{ libs.}$$

When the beam is uniformly loaded throughout its length, the result must be doubled, *i. e.* it will support double the weight.

When the beam is fixed at both ends and loaded in the middle, one-half of the result must be added; and if the weight is laid uniformly along its length, the result must be tripled.

These Problems are taken from Barlow's Essay, as before quoted: he, however, gives a second Rule to each of the Problems, in which the angle of deflection is considered. These Rules give higher results than those here stated; but for practice the first Rule is the best, being more simple and safe.

It is considered that two-thirds of the result is sufficient to lay upon a beam for a permanent load.

PRACTICAL PROBLEMS *for the Transverse strength of Cast Iron Beams.*[*]

PROBLEM I.

To find the breadth of a uniform Cast Iron Beam to bear a given weight in the middle.

RULE 1. Multiply the length of bearing in feet, or the length between the supports, by the weight to be supported in libs; and divide this product by 850 times the square of the depth in inches; the quotient will be the breadth in inches required.

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[*] See Tredgold's Practical Essay on the Strength of Cast Iron, p. 80.

RULE 2. Multiply the length of bearing in feet, by the weight to be supported in libs, and divide this product by 850 times the breadth in inches; and the square root of the quotient will be the depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the n^{th} part of the depth, n representing any number at will. Then the Rule is as follows:—

Multiply n times the length in feet, by the weight in libs; divide this product by 850, and the cube root of the quotient will be the depth required; and the breadth will be the n^{th} part of the depth.

Note. It may be remarked here, that the Rules are the same for inclined as for horizontal beams, when the horizontal distance between the supports is taken for the length of bearing.

EXAMPLE I.

What is the breadth of a Beam 20 feet long, 15 inches deep, and to be loaded with 13 tons?

$$13 \text{ tons} = 29120 \text{ libs.}$$

$$\frac{29120 \times 20}{15^2 \times 850} = 3.045 \text{ inches broad.}$$

EXAMPLE II.

What is the depth of a Beam 20 feet long, 3 inches broad, and to support a weight of 13 tons?

$\frac{29120 \times 20}{850 \times 3} = 225$, the square root of which is = 15 inches, the depth required.

EXAMPLE III.

What are the cross sectional dimensions of a Beam 30 feet long, and of sufficient strength to support a weight of 10 tons; the depth being twice the breadth?——n will therefore be = 2

10 tons = 22400 libs. Length = 30 $30 \times \overset{n}{2} = 60$

$\frac{22400 \times 60}{850} = 1581$, the cube root of which is nearly 11½, which is equal to the depth in inches: the breadth is the half of the depth = 5¾ inches.

PROBLEM II.

To find the breadth, when the load is not in the middle between the supports.

Rule. Multiply the short length by the long length, and four times this product divided by the whole length between the supports, will give the effective leverage of the load in feet; this quotient being used instead of the length, in any of the Rules in the foregoing Problem, the breadth and depth will be found by them.

EXAMPLE.

What are the cross sectional dimensions of a Beam 12 feet long, supporting a weight of 15 tons, 3 feet from the one end, when the breadth is a fourth of the depth?

$$\frac{3\times9\times4}{12}=9 \quad 9\times\overset{n}{4}=36 \quad 15 \text{ tons}=33600 \text{ libs.}$$

$\frac{33600\ \times\ 36}{850} = 1423$, the cube root of which is $= 11\frac{1}{4}$, the depth: the breadth will be $\frac{11\frac{1}{4}}{4}=2\frac{13}{16}$

PROBLEM III.

To find the breadth when the load is uniformly distributed over the length of the beam.

Rule. The same Rules apply as in Prob. 1, only the divisor is changed from 850 to 1700, *i. e.* when the load is uniformly distributed over the length of the beam, it supports double the weight than when the load is laid on the middle.

Note. Examples in Problem 1. apply to this Problem, only changing the divisors, or halving the quotients.

PROBLEM IV.

To find the dimensions, when a beam is fixed at one end and loaded at the other, or when it is supported at the middle and loaded at both ends.

Rule. Take the horizontal length of the projection of the beam, when fixed at one end, for the length, and apply the Rules in Prob. 1. only using the divisor 212 instead of 850.

When the beam is supported any where between the two ends, multiply the length from the prop by the weight hung at the end, and apply the remainder of the Rule as in Prob. 1. only using 212 for 850.

When the load is uniformly distributed over the length of the projection, employ 425 instead of 212 as a divisor.

Note. The Rules of this Problem apply to the teeth of wheels, the length being the length of the teeth, and the depth, the thickness of the teeth.

Example to this Note.

Let the greatest power acting at the pitch line of the wheel be 6000 libs, and the thickness of the teeth 1½ inch, the length of the teeth being ¼ foot; What is the breadth of the teeth?

$$\frac{6000 \times .25}{212 \times \overline{1.5}^2} = \frac{1500}{477} = 3.14 \text{ inches the breadth;}$$

but to allow for wearing by friction, this quotient is doubled, or 6½ inches = the breadth of the teeth, or face of the wheel.

PROBLEM V.

To find the diameter of a solid cylinder to support a given weight in the middle—between the middle and the end—and when the weight is uniformly distributed over the length—also when fixed at one end.

When the weight is placed in the middle.

RULE. Multiply the weight in libs by the length in feet; divide this product by 500, and the cube root of the quotient will be the diameter in inches.

When the weight is between the middle and the end.

Rule. Multiply the short end by the long end; then multiply that product by 4 times the weight in libs. Divide this product by 500 times the length in feet, and the cube root of the quotient will be the diameter in inches.

When the load is uniformly distributed over the length.

Rule. Multiply the length in feet by the weight in libs, and one-tenth of the cube root of the product will be the diameter in inches.

When fixed at one end, and the load applied at the other.

Rule. Multiply the length of projection in feet by the weight in libs, and the 5th part of the cube root of this product will be the diameter in inches.

The Rules for the deflection of Beams and Bars are here omitted, being considered, that, in most of practical cases, the deflection is of little importance; however, when it is of importance, reference to Barlow's Essay on the strength of Timber, and Tredgold's Essay on the strength of Iron, will satisfy all inquiries. These books ought to be in the possession of every Mechanic, as they give the most comprehensive and most correct data, for the strength of materials, of any that have yet appeared.

STRENGTH OF THE JOURNALS OF SHAFTS.

LATERAL STRENGTH.

THE Rules in Problem 5. of last article, can be here applied. Mr. Robertson Buchanan, in his Essay on the Strength of Shafts, uses the following Rule, which is simple enough, and easy to be remembered; but the above-mentioned Rules are the most correct, and ought to be used on all occasions.

Mr. Buchanan's Rule is,—"The cube root of the weight in cwts. is nearly equal to the diameter of the Journal."—"*Nearly equal,*—being prudent to make the Journal a little more than less, and to make a due allowance for wearing."

EXAMPLE.

What is the diameter of the Journal of a Water Wheel Shaft, 13 feet long, the weight of the Wheel being 15 tons?

By Mr. B.'s Rule. $\sqrt[3]{15\times20}=6.7$ or 7 inches diam$^{r.}$

By Mr. Tredgold's Rule.

Weight in the middle. $\left\{ \frac{33600\times13}{500}=873 \quad \sqrt[3]{873}=9\frac{1}{2} \right.$ inches diam$^{r.}$

Weight equally distributed. $\left\{ 33600\times13=436800 \quad \frac{\sqrt[3]{436800}}{10}=7.65 \right.$ In.

TO RESIST TORSION OR TWISTING.

It is obvious that the strength of revolving Shafts are directly as the cubes of their diameters and revolutions; and inversely, as the resistance they have to overcome.

Mr. Robertson Buchanan, in his Essay on the strength of Shafts, gives the following data, deduced from several experiments, viz. That the Fly Wheel Shaft of a 50 horse power engine, at 50 revolutions per minute, requires to be 7½ inches diameter, and therefore, the cube of this diameter, which is = 421.875, serves as a multiplier to all other shafts in the same proportion; and taking this as a standard, he gives the following Multipliers, viz.

For the Shaft of a Steam Engine, Water Wheel, or any Shaft connected with a first power, - - - - - -	400
For Shafts in inside of Mills, to drive smaller Machinery, or connected with the Shafts above, - - - - - - -	200
For the small Shafts of a Mill or Machinery,	100

From the foregoing, the following Rule is derived, viz.

The number of horses' power a Shaft is equal to, is directly as the cube of the diameter and number of revolutions; and inversely, as the above Multipliers.

Note. Shafts here are understood as the Journals of Shafts, the bodies of Shafts being generally made square.

EXAMPLE I.

When the Fly Wheel Shaft of a 45 horse power Steam Engine makes 90 revolutions per minute, what is the diameter of the Journal?

$$\frac{45 \times 400}{90} = 200 \quad \sqrt[3]{200} = 5\tfrac{8}{10} \text{ inches diameter.}$$

EXAMPLE II.

The velocity of a Shaft is 80 revolutions per minute, and its diameter is 3 inches: What is its power?

$$\frac{3^3 \times 80}{400} = 5.4 \text{ horse power.}$$

EXAMPLE III.

What will be the diameter of the Shaft in the first Example, when used as a Shaft of the second Multiplier?*

$$\frac{5.8}{1.25} = 4.64, \text{ or } \sqrt[3]{\frac{45 \times 200}{90}} = 4\tfrac{6}{10} \text{ inches diameter.}$$

The following is a Table of the diameters of Shafts, being the First Movers, or having 400 for their multipliers.

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* The diameters of the second movers will be found by dividing the numbers in the Table by 1.25, and the diameters of the third movers, by dividing the numbers by 1.56.

TABLE.

DIAMETERS OF THE JOURNALS OF FIRST MOVERS.

Horses' Power.	REVOLUTIONS.									
	10	15	20	25	30	35	40	45	50	55
4	5.5	4.8	4.5	4.	3.7	3.8	3.5	3.3	3.2	3.1
5	5.9	5.1	4.7	4.4	4.1	3.9	3.7	3.6	3.5	3.3
6	6.3	5.5	5.	4.6	4.4	4.1	4.	3.8	3.7	3.6
7	6.6	5.8	5.2	4.9	4.6	4.4	4.2	4.	3.9	3.7
8	6.9	6.	5.5	5.1	4.8	4.6	4.4	4.2	4.1	4.
9	7.2	6.3	5.7	5.5	5.	4.8	4.5	4.4	4.2	4.1
10	7.4	6.6	5.9	5.6	5.2	4.9	4.7	4.6	4.4	4.2
12	7.9	6.9	6.3	5.8	5.6	5.4	5.2	5.	4.8	4.6
14	8.3	7.2	6.7	6.2	5.9	5.6	5.4	5.2	5.	4.7
16	8.7	7.6	7.1	6.6	6.1	5.8	5.6	5.4	5.2	5.
18	9.	7.9	7.5	7.	6.6	6.2	5.8	5.6	5.4	5.2
20	9.3	8.1	7.4	7.2	6.6	6.4	5.9	5.7	5.6	5.4
25	10.	8.5	8.	7.4	7.1	6.8	6.3	6.	5.9	5.6
30	10.7	9.3	8.4	7.9	7.4	7.1	6.9	6.7	6.5	6.3
35	11.4	9.8	8.9	8.4	7.9	7.4	7.1	6.9	6.6	6.5
40	11.7	10.5	9.3	8.8	8.3	7.8	7.4	7.2	6.9	6.7
45	12.	10.6	9.7	9.2	8.7	8.1	7.6	7.4	7.	6.8
50	12.6	11.	10.	9.3	9.	8.5	8.	7.8	7.4	7.3
55	13.4	11.4	10.4	9.8	9.1	8.8	8.4	8.	7.5	7.4
60	13.6	12.	10.8	10.	9.3	9.	8.6	8.2	7.7	7.6

INCHES DIAMETER.

TABLE CONTINUED.

Horses' Power.	REVOLUTIONS.									
	60	65	70	75	80	85	90	95	100	105
4	3.	2.9	2.9	2.8	2.7	2.7	2.6	2.6	2.6	2.5
5	3.3	3.2	3.1	3.	3.	2.9	2.9	2.8	2.8	2.7
6	3.5	3.5	3.4	3.3	3.2	3.2	3.	3.	2.9	2.9
7	3.6	3.6	3.5	3.4	3.4	3.3	3.3	3.2	3.1	3.1
8	3.9	3.8	3.7	3.6	3.5	3.5	3.4	3.4	3.3	3.2
9	4.	3.8	3.7	3.7	3.6	3.6	3.5	3.5	3.4	3.3
10	4.1	4.	3.9	3.8	3.7	3.7	3.6	3.6	3.5	3.4
12	4.4	4.3	4.2	4.1	4.	3.9	3.8	3.8	3.7	3.6
14	4.5	4.4	4.4	4.3	4.2	4.1	4.	4.	3.9	3.8
16	4.8	4.7	4.6	4.5	4.4	4.4	4.3	4.2	4.1	4.
18	5.	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.2
20	5.2	5.1	5.	4.8	4.6	4.6	4.5	4.5	4.4	4.4
25	5.5	5.4	5.3	5.2	5.1	4.9	4.8	4.7	4.6	4.6
30	5.9	5.8	5.7	5.6	5.5	5.3	5.2	5.1	5.	4.9
35	6.3	6.1	5.9	5.7	5.6	5.5	5.4	5.3	5.2	5.2
40	6.6	6.4	6.2	6.	5.9	5.8	5.7	5.6	5.6	5.5
45	6.7	6.5	6.4	6.2	6.1	6.	5.9	5.8	5.7	5.6
50	7.2	6.9	6.8	6.6	6.5	6.4	6.2	6.	5.9	5.8
55	7.3	7.2	7.	6.7	6.6	6.5	6.3	6.2	6.1	6.
60	7.4	7.3	7.2	6.9	6.8	6.8	6.7	6.6	6.4	6.2
	INCHES DIAMETER.									

It is a well known fact, that a cast iron rod will sustain more torsional pressure, than a malleable iron rod of the same dimensions.—That is, a malleable iron rod will be twisted by a less weight than what is required to wrench a cast iron rod of the same dimensions.

When the strength of malleable iron is less than that of cast iron to resist torsion; it is stronger than cast iron to resist lateral pressure, and that strength is in proportion as 9 is to 14.

From the foregoing, it is easy for the Mill-wright to make his Shafts of the iron best suited to overcome the resistance to which they will be subject, and the proportion of the diameters of their Journals, according to the iron of which they are made:—for example; What will be the diameter of a malleable iron Journal, to sustain an equal weight with a cast iron Journal of 7 inches diameter?

$$7^3 = 343$$

$14 : 343 :: 9 : 220\frac{1}{2}$ now $\sqrt[3]{220.5} = 6.04$ inches diameter.

STRENGTH OF WHEELS.

THE arms of Wheels are as levers fixed at one end and loaded at the other, and consequently the greatest strain is upon the end of the arm next the axle; for that reason all arms of wheels should be strongest at that part, and tapering towards the rim.

The Rule for the breadth and thickness of arms, according to their length and number in the wheel, is as follows: *(See Tredgold's Essay, page* 114.) Multiply the power or weight acting at the end of the arm by the cube of its length; the product of which, divided by 2656 times the number of arms multiplied by the deflection, will give the breadth and cube of the depth.

EXAMPLE.

Suppose the force acting at the circumference of a spur wheel to be 1600 libs, the radius of wheel 6 feet, and number of arms 8, and let the deflection not exceed $\frac{1}{10}$ of an inch.

$$\frac{1600 \times 6^3}{2656 \times 8 \times .1} = 163 = \text{breadth and cube of the depth.}$$

Let the breadth be 2.5 inches, therefore $\frac{163}{2.5} = 65.2$, which is equal to the cube of the depth: now the cube root of 65.2 is nearly 4.03 inches; this, consequently, is the depth, or dimension, of each arm in the direction of the force.

Note. When the depth at the rim is intended to be half that of the axes, use 1640 as a divisor instead of 2656.

The teeth are as beams, or cantilevers, fixed at one end and loaded at the other, the Rule applying direct to them (*See Tredgold's Essay, Art.* 121) where the length of the beam is the length of the teeth, and the depth, the thickness of the teeth. For the better explanation of the Rule the following Example is given.

EXAMPLE.

The greatest power acting at the pitch line of the wheel is 6000 libs, and the thickness of the teeth $1\frac{1}{2}$ inch, the length of the teeth being 0.25 feet; it is required to determine the breadth of the teeth?

$$\frac{6000 \times 0.25}{212 \times 1.5^2} = \frac{1500}{477} = 3.2 \text{ inches the breadth}$$

required.

In order that the teeth may be capable of offering a sufficient resistance after being worn by friction, the breadth thus found should be doubled; therefore, in the above Example, the breadth should be 6.4, or say $6\frac{1}{2}$ inches.

Mr. Carmichael[*] gives the following data, gleaned from experiments, which is therefore valuable, and of much use to the practical mechanic.

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[*] See Robertson Buchanan on the Teeth of Wheels.

Rule. Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length; the quotient will be the proportional strength in horses' power, with a velocity of 2.27 feet per second.

EXAMPLE.

What is the power of a wheel, the teeth of which are 6 inches broad, 1.5 inch thick, and 1.8 inch long, and revolving at the velocity of 3 feet per second?

$$\frac{\overline{1.5^2 \times 6}}{1.8} = \frac{13.5}{1.8} = 7.5 \text{ strength at 2.27 feet per sec.}$$

$$\text{then } 2.27 : 7.5 :: 3 = \frac{7.5 \times 3}{2.27} = 9.91 \text{ horse-power.}$$

Rule. The pitch is found by multiplying the thickness by 2.1, and the length is found by multiplying the thickness by 1.2.

EXAMPLE.

The thickness being 2 inches, what is the pitch and length?

$$2 \times 2.1 = 4.2 \text{ Pitch.}$$
$$2 \times 1.2 \times 2.4 \text{ Length.}$$

Note. The breadth of the teeth, as commonly executed by the best Masters, seems to be from about twice to thrice the pitch.

TABLE.

Pitch in Inches.	Thickness in Inches.	Breadth in Inches.	Length in Inches.	Horses Power at 2.27 feet ⅌ Second.	H. P. at 3 feet ⅌ Second.	H. P. at 6 feet ⅌ Second.	H. P. at 11 feet ⅌ Second.
4.2	2.	8.	2.40	13.33	17.61	35.23	64.6
3.99	1.9	7.6	2.28	13.03	15.90	31.80	58.30
3.78	1.8	7.2	2.16	10.80	14.27	28.54	52.32
3.57	1.7	6.8	2.04	9.63	12.72	25.54	46.68
3.36	1.6	6.4	1.92	8.53	11.27	22.54	41.32
3.15	1.5	6.	1.80	7.50	9.91	19.82	36.33
2.94	1.4	5.6	1.68	6.53	8.63	17.26	31.64
2.73	1.3	5.2	1.56	5.63	7.44	14.88	27.28
2.52	1.2	4.8	1.44	4.80	6.34	12.68	23.24
2.31	1.1	4.4	1.32	4.03	5.32	10.64	19.54
2.10	1.	4.	1.20	3.33	4.40	8.81	16.15
1.89	.9	3.6	1.08	2.70	3.57	7.14	13.09
1.68	.8	3.2	.96	2.13	2.81	5.62	10.33
1.47	.7	2.8	.84	1.63	2.15	4.30	7.88
1.26	.6	2.4	.72	1.20	1.59	3.18	5.83
1.05	.5	2.	.60	.83	1.10	2.20	4.03

VELOCITY OF WHEELS.

WHEELS are for conveying motion to the different parts of a machine, at the same, or at a greater or less velocity, as may be required.—When two wheels are in motion their teeth act on one another alternately, and consequently, if one of these wheels has 40 teeth, and the other 20 teeth, the one with 20 will turn twice upon its axis, for one revolution of the wheel with 40 teeth.—From this the Rule is taken, which is,—As the velocity required is to the number of teeth in the driver, so is the velocity of the driver to the number of teeth in the driven.

Note. To find the proportion that the velocities of the wheels in a train should bear to one another, subtract the less velocity from the greater, and divide the remainder by the number of one less than the wheels in the train; the quotient will be the number rising in Arithmetical progression, from the least to the greatest velocity of the train of wheels.

EXAMPLE I.

What is the number of teeth in each of three wheels to produce 17 revolutions per minute, the driver having 107 teeth, and making 3 revolutions per minute?

$\frac{17 - 3 = 14}{3 - 1 = 2} = 7$, therefore 3 10 17 are the velocities of the three wheels.

$$\text{By the Rule.}\begin{cases} 10:107::\ 3:32=\dfrac{107\times 3}{10}=32\text{ teeth.}\\ 17:\ 32::10:19=\dfrac{32\times 10}{17}=19\text{ teeth.}\end{cases}$$

EXAMPLE II.

What is the number of teeth in each of 7 wheels, to produce 1 revolution per minute, the driver having 25 teeth, and making 56 revolutions per minute?

$\frac{56 - 1 = 55}{7 - 1 = 6} = 9$, therefore 56 46 37 28 19 10 1, are the progressional velocities.

46 : 25 :: 56 : 30 Teeth.
37 : 30 :: 46 : 37 ——
28 : 37 :: 37 : 49 ——
19 : 49 :: 28 : 72 ——
10 : 72 :: 19 : 137 ——
1 : 137 :: 10 : 1370 ——

It will be observed that the last wheel, in the foregoing Example, is of a size too great for application;

to obviate this difficulty, which frequently arises in this kind of training, wheels and pinions are used, which give a great command of velocity.—Suppose the velocities of last Example, and the train only of 2 wheels and 2 pinions.

$\dfrac{56 - 1 = 55}{4 - 1 = 3} = 18$, therefore 56 19 1, are the progressional velocities.

19 : 25 : : 56 : 74 = teeth in the wheel driven by the first driver, and 1 : 10 : : 19 : 190 = teeth, in the second driven wheel, 10 teeth being in the driving pinion. - 25 drivers 74 driven.
10 —— 190 ——

The following is a Table of the Radii of Wheels, from ten to three hundred teeth, the pitch being 2 inches.

The radius for any other pitch may be found by the following analogy:—As two Inches is to the Radius in the Table, so is the new Pitch to the new Radius.

TABLE.

No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.
10	3.236	46	14.654	82	26.108	118	37.565
11	3.549	47	14.972	83	26.426	119	37.883
12	3.864	48	15.290	84	26.741	120	38.202
13	4.179	49	15.608	85	27.063	121	38.520
14	4.494	50	15.926	86	27.381	122	38.838
15	4.810	51	16.244	87	27.699	123	39.156
16	5.126	52	16.562	88	28.017	124	39.475
17	5.442	53	16.880	89	28.336	125	39.793
18	5.759	54	17.198	90	28.654	126	40.111
19	6.076	55	17.517	91	28.972	127	40.429
20	6.392	56	17.835	92	29.290	128	40.748
21	6.710	57	18.153	93	29.608	129	41.066
22	7.027	58	18.471	94	29.927	130	41.384
23	7.344	59	18.789	95	30.245	131	41.703
24	7.661	60	19.107	96	30.563	132	42.021
25	7.979	61	19.425	97	30.881	133	42.339
26	8.296	62	19.744	98	31.200	134	42.657
27	8.614	63	20.062	99	31.518	135	42.976
28	8.931	64	20.380	100	31.836	136	43.294
29	9.249	65	20.698	101	32.155	137	43.612
30	9.567	66	21.016	102	32.473	138	43.931
31	9.885	67	21.335	103	32.791	139	44.249
32	10.202	68	21.653	104	33.109	140	44.567
33	10.520	69	21.971	105	33.427	141	44.885
34	10.838	70	22.289	106	33.746	142	45.204
35	11.156	71	22.607	107	34.064	143	45.522
36	11.474	72	22.926	108	34.382	144	45.840
37	11.792	73	23.244	109	34.700	145	46.158
38	12.110	74	23.562	110	35.018	146	46.477
39	12.428	75	23.880	111	35.337	147	46.795
40	12.746	76	24.198	112	35.655	148	47.113
41	13.064	77	24.517	113	35.974	149	47.432
42	13.382	78	24.835	114	36.292	150	47.750
43	13.700	79	25.153	115	36.611	151	48.068
44	14.018	80	25.471	116	36.929	152	48.387
45	14.336	81	25.790	117	37.247	153	48.705

TABLE CONTINUED.

No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.
154	49.023	191	60.800	228	72.577	265	84.354
155	49.341	192	61.118	229	72.895	266	84.673
156	49.660	193	61.436	230	73.214	267	84.991
157	49.978	194	61.755	231	73.532	268	85.309
158	50.296	195	62.073	232	73.850	269	85.627
159	50.615	196	62.392	233	74.168	270	85.946
160	50.933	197	62.710	234	74.487	271	86.265
161	51.251	198	63.028	235	74.805	272	86.582
162	51.569	199	63.346	236	75.123	273	86.900
163	51.888	200	63.665	237	75.441	274	87.219
164	52.206	201	63.983	238	75.760	275	87.537
165	52.524	202	64.301	239	76.078	276	87.855
166	52.843	203	64.620	240	76.397	277	88.174
167	53.161	204	64.938	241	76.715	278	88 462
168	53.479	205	65.256	242	77.033	279	88.810
169	53.798	206	65.574	243	77.351	280	89.129
170	54.116	207	65.893	244	77.670	281	89.447
171	54.434	208	66.211	245	77.988	282	89.765
172	54.752	209	66.529	246	78.306	283	90.084
173	55.071	210	66.848	247	78.625	284	90.402
174	55.389	211	67.166	248	78.943	285	90.720
175	55.707	212	67.484	249	79.261	286	91.038
176	55.026	213	67.803	250	79.580	287	91.357
177	55.344	214	68.121	251	79.898	288	91.675
178	56.662	215	68.439	252	80.216	289	91.993
179	56.980	216	68.757	253	80.534	290	92.312
180	57.299	217	69.075	254	80.853	291	92.630
181	57.617	218	69 394	255	81.171	292	92.948
182	57.935	219	69.712	256	81.489	293	93.267
183	58.253	220	70.031	257	81.808	294	93.585
184	58.572	221	70.349	258	82.126	295	93.903
185	58.890	222	70.667	259	82.444	296	94.222
186	59.209	223	70.985	260	82.763	297	94.540
187	59.527	224	71.304	261	83.081	298	94.858
188	59.845	225	71.622	262	83.399	299	95.177
189	60.163	226	71.941	263	83.717	300	95.495
190	60.482	227	72.258	264	84.038		

CENTRE OF GRAVITY.

THE centre of Gravity of a body is that point, which, if sustained, the body remains at rest; the particles of which it is composed being equipoised, and having their weights collected, as it were, into that point.

Bodies are reciprocal to each other as their distances from the centre of gravity.—Suppose a rod 11 inches long, with a weight of 2 libs hung at the one end, and a weight of 20 libs hung at the other end, the centre of gravity, or the point on which this rod so loaded, will balance itself, is just 1 inch from the greater weight, and 10 inches from the less, because, $20 \times 1 = 20$, and $2 \times 10 = 20$, therefore, their weights are inversely as their distances from the centre of gravity.—Hence, the method to find the common centre of gravity of any number of bodies, is, first find the centre between two bodies, then the centre between that centre and a third body, and so on for a fourth, fifth, &c.; the last centre found being the common centre of all the bodies.

From the foregoing it will easily be conceived, that if a homogeneous beam is balanced upon a point, that point will be the centre of gravity, and also the centre of the beam; but suppose the beam 10 feet long, each foot weighing 8 libs, and a weight of 90 libs suspended from the one end, at what point of the beam will the centre of gravity be?

10 feet, length of beam.—8 libs each foot in length.
90 libs weight suspended.

$$\frac{8 \times 10 + \overline{2 \times 90}}{8 \times 10 + 90} \times \frac{10}{2} = \frac{260}{170} \times 5 = 7.65$$

$+ 2.35 = 10$ feet length of beam, that is, the centre of gravity is 2.35 feet from the end at which the weight of 90 libs is suspended, and will be 7.65 feet from the other end.

Suppose another homogeneous beam, 12 feet long, with a weight of 100 libs fixed at one end, it is found that the whole is in equilibrio when the beam is suspended 2 feet from the end next the weight; what is the weight of the beam?

100 libs weight suspended.
2 feet distance from the weight.
10 feet distance from the other end.

$$\frac{2 \times 100 \times 2}{100 - 4} = \frac{400}{96} = 4.166$$ libs the weight of 1 foot of beam, and $4.166 \times 12 = 49.992$ libs, the weight of the beam.

It is well known to every practical Mechanic, that there are no homogeneous beams or bars:—that it is impossible to find the weight of a foot of length, in a piece of wood, iron, stone, &c. and that the exact centre of gravity of such materials cannot be found by any known theorem. To obviate this difficulty, and ascertain the true centre of gravity, the beams, bars, &c. are balanced over a prop; but there are many large unwieldy bodies that cannot be thus treated, and for this reason the following data are given, which ascertain nearly the centre required; the data being taken, which are nearest the form and distribution of weight over the body, of which the centre of gravity is required.

1. The centre of gravity of a triangle is in the straight line, drawn from any angle to the bisection of the opposite side, at the distance of $\frac{2}{3}$ of that line from the angle.

This rule is also true with regard to a pyramid of any number of sides; also to a cone.

2. The centre of gravity of a segment of a circle, is in the radius which bisects it; and its distance from the centre of the circle, is $\frac{1}{12}$ of the cube of its cord divided by the area of the segment.

3. The centre of gravity of a sector of a circle is in the radius which bisects it; and its distance from the centre of the circle, is a fourth proportional to the arc, its chord, and $\frac{2}{3}$ of the radius.

For further information in this article, *see Hutton's Mathemathics, Banks on Mills, Venturoli's Mechanics by Cresswell, &c.*

CENTRE OF PERCUSSION.

THE centre of Oscillation or Percussion, is the point in a body revolving round a fixed axis, so taken, that when it is stopped by any force, the whole motion, and tendency to motion, of the revolving body, is stopped at the same time.

It is also that point of a revolving body, which would strike any obstacle with the greatest effect; and from this property, it has received the name of percussion.

The centres of oscillation and percussion are generally treated separately; but the two centres are in the same point, and therefore their properties are the same.

As in bodies at rest, the whole weight may be considered as collected in the centre of gravity; so in bodies in motion, the whole force may be considered as concentrated in the centre of percussion:—therefore, the weight of the rod multiplied by the distance of the centre of gravity from the point of suspension, will be equal to the force of the rod divided by the distance of the centre of percussion from the same point. For example, the length of a rod being 20 feet, and the weight of a foot in length equal to 100 oz.; also a weight or ball fixed at under end weighing 1000 oz.; at what point of the rod from the point of suspension will be the centre of percussion?*

The weight of the rod is $20 \times 100 = 2000$ oz. which multiplied by half its length $2000 \times 10 = 20000$, gives the momentum of the rod. The weight of the ball $= 1000$ oz. multiplied by the length of rod $= 1000 \times 20$, gives the momentum of the ball. Now the weight of the rod multiplied by the square of the length, and divided by $3 = \frac{2000 \times 20^2}{3} = 266666$, = the force of the rod, and the weight of the ball multiplied by the square of the length of the rod, $1000 \times 20^2 = 400000$, is the force of the ball:—therefore the centre of percussion $= \frac{266666 + 400000}{20000 + 20000} = \frac{666666}{40000} = 16.66$ feet.

For another example; suppose a rod 12 feet long, and 2 lbs. each foot in length, with 2 balls of 3 lbs. each, one fixed 6 feet from the point of suspension, and the other at the end of the rod; what is the distance between the points of suspension and percussion?

$12 \times 2 \times 6 = 144$ momentum of rod.
$3 \times 6 = 18$ do. of 1st. ball.
$3 \times 12 = 36$ do. of 2d. do.
$\overline{198}$

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*$a = 20$ feet long.
$b = 100$ oz. wt. of a foot in length.
$c = 1000$ do. fixed at end.

$$\left.\begin{array}{l} a = 20 \text{ feet long.} \\ b = 100 \text{ oz. wt. of a foot in length.} \\ c = 1000 \text{ do. fixed at end.} \end{array}\right\} \frac{\frac{1}{3}ab \times a^2 + ca^2}{\frac{1}{2}ab \times a + ac} = \text{centre of percussion.}$$

$$\begin{array}{rrl} \frac{24 \times 144}{3} = & 1152, & \text{force of rod.} \\ 3 \times 36 = & 108 & \text{do. of 1st. ball.} \\ 3 \times 144 = & \underline{432} & \text{do. of 2d. ball.} \\ & 1692 & \end{array}$$

therefore the centre of percussion $= \frac{1692}{198} = 8.5\dot{4}\dot{5}$ feet from the point of suspension.

As the centre of percussion is the same with the centre of gravity in the non-application to practical purposes, the following is the easiest and simplest mode of finding it in any beam, bar, &c.

Suspend the body very freely by a fixed point, and make it vibrate in small arcs, counting the number of vibrations it makes in any time, as a minute, and let the number of vibrations made in a minute be called n; then shall the distance of the centre of oscillation from the point of suspension be $\frac{140850}{n^2} =$ inches.—For the length of the pendulum vibrating seconds, or 60 times in a minute, being $39\frac{1}{8}$ inches; and the lengths of the pendulums being reciprocally as the square of the number of vibrations made in the same time:—therefore, $n^2 : 60^2 :: 39\frac{1}{8} : \frac{60^2 \times 39\frac{1}{8}}{n^2} = \frac{140850}{n^2}$ being the length of the pendulum which vibrates n times in a minute, or the distance of the centre of oscillation belowthe axis of motion.

There are many situations in which bodies are placed, that prevent the application of the above rule; and for this reason the following data are given, which will be found useful when the bodies and the forms here given correspond.

1. If the body is a heavy straight line of uniform density, and is suspended by one extremity, the distance of its centre of percussion is ⅔ds of its length.

2. In a slender rod of a cylindrical or prismatic shape, whose breadth is very small compared with its length, has the distance of its centre of percussion nearly ⅔ds of its length from the axis of suspension.

If these rods were formed so that all the points of their transverse sections were equidistant from the axis of suspension, the distance of the centre of percussion would be exactly ⅔ds of their length.

3. In an Isosceles Triangle, suspended by its apex, and vibrating in a plane perpendicular to itself, the distance of the centre of percussion is ¾ths of its altitude. A line or rod, whose density varies as the distance from its extremity, or the point of suspension; *also a fly-wheel, or wheels in general*, is in precisely the same predicament as the Isosceles Triangle; *i. e.* the centre of percussion is distant from the centre of suspension ¾ths of its length.

4. In a very slender cone or pyramid, vibrating about its apex, the distance of its centre of percussion is nearly ⅘ths of its length.

See Edinburgh Ency.; Article, Centre of Percussion.

CENTRE OF GYRATION.

THE centre of Oscillation already described, is the point into which all the matter of a body is collected, when the body is put in motion by its own gravity; and the centre of Gyration is the point into which all the matter of a body is collected, when it is put in motion by any extraneous force.

If a straight bar, equally thick, was struck at the centre of gyration, the stroke would communicate the same angular velocity to the bar, as if the whole bar was collected in that point.

The force of any particle revolving round a centre, is, as that particle multiplied by the square of its velocity, or of its distance from the centre of motion; consequently, the force required to destroy that motion must be equal to it.

For example; suppose a bar of a uniform density 12 feet long, and each foot weighing 7 libs, and revolving upon a centre 3 feet from the one end; at what distance will the centre of gyration be from the centre of motion?

$a = 9$ feet long end.
$b = 3$ do. short end.
$c = 7$ libs each foot.

$9 \times 7 = 63$, weight of long end.
$3 \times 7 = 21$, weight of short end.
$63 \times 9^2 = 5103$, force of long end.
$21 \times 3^2 = 189$, do. of short end.

Centre of gyration $\sqrt{\frac{5103+189}{3\times\overline{63+21}}}$ = 4.5 feet from centre of motion.*

For another example—Let the same bar have a weight of 50 libs at each end, then at what distance will the centre of gyration be from the centre of motion?

a = 9 feet, long end.
b = 3 do. short end.
c = 7 libs each foot.
d = 50 libs weight at long end.
e = 50 libs weight at short end.
z = distance of centre of gyration from centre of motion.

$63 \times 9^2 = 5103$, force of long end.
$21 \times 3^2 = 189$, do. of short end.
$50 \times 9^2 = 4050$, do. of weight on long end.
$50 \times 3^2 = 450$, do. of weight on short end.

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*$\frac{ac.a^2}{3}+\frac{bc.b^2}{3}=\overline{ac+bc}\times z^2$ or $z=\sqrt{\frac{ac.a^2+bc.b^2}{3\,\overline{ac+bc}}}$ z being the centre of gyration.

$$\sqrt{\frac{3 \times \overline{4050 + 450 + 5103 + 189}}{3 \times \overline{50 + 50 + 63 + 21}}} = \sqrt{\frac{18792}{552}} = 5.84$$

distance between the centres.*

The centre of gyration, with respect to practical utility, is the same as the two foregoing centres. The following Rule is the easiest mode of ascertaining the centre of gyration.

"If the distance of the centre of oscillation from the centre of the system, or point of suspension, be multiplied by the distance of the centre of gravity from the same point, the square root of the product will be the distance of the centre of gyration, *i. e.* let the centre of gravity be 4, and the centre of oscillation 9, then 4 × 9 = 36, and the square root of that is 6; therefore 6 is the distance that the centre of gyration is from the point of suspension."

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* $da^2 + \frac{ca^3}{3} + eb^2 + \frac{cb^3}{3}$ = the force of the whole revolving round the centre of motion, and which must be equal to $\overline{ac + d + bc + e} \times Z^2$, therefore $Z^2 \times \overline{ac + d + bc + e} = da^2 + eb^2 + \frac{ca^3 + cb^3}{3}$, or

$$Z^2 = \frac{3\overline{da^2 + eb^2} + ca^3 + cb^3}{3\overline{ac + d + bc + e}}, \text{ and}$$

$$Z = \sqrt{\frac{3da^2 + eb^2 + ca^3 + cb^3}{3ac + d + bc + e}}.$$

The following note is given by Dr. Hutton, in the 3d vol. of his Math. art. max. of Machines, p. 244.

The distance of R, the centre of gyration, from C the centre or axis of motion, in some of the most useful cases, is as below.

In a circular wheel of uniform thickness	$CR = \text{rad.}\sqrt{\frac{1}{2}}$.
In the periphery of a circle revolving about the diameter .	$CR = \text{rad.}\sqrt{\frac{1}{2}}$.
In the plane of a circle ditto .	$CR = \frac{1}{2}\,\text{rad.}$
In the surface of a sphere ditto .	$CR = \text{rad.}\sqrt{\frac{2}{3}}$.
In a solid sphere . . ditto .	$CR = \text{rad.}\sqrt{\frac{3}{5}}$.
In a plane ring formed of circles whose radii are R, r, revolving about centre	$CR = \sqrt{\frac{R^4}{2R^2 - 2r^2}}$.
In a cone revolving about its vertex	$CR = \frac{1}{2}\sqrt{\frac{12}{5}a^2 + \frac{3}{5}r^2}$.
In a cone . . . . its axis .	$CR = r\sqrt{\frac{3}{15}}$.
In a straight lever whose arms are R and r	$CR = \sqrt{\frac{R^3 + r^3}{3(R+r)}}$.

SUMMARY.

If P be any particle of a body B, and d its distance from the axis of motion S, also G O R the centres of Gravity, Oscillation, and Gyration. Then the centres of

$$\text{Gravity will be} = \frac{Pd}{B} = G.$$

$$\text{Percussion do.} = \frac{Pd^2}{SGB} = O.$$

$$\text{Gyration do.} = \sqrt{\frac{Pd^2}{B}} = R.$$

For ample Explanations and Examples of the foregoing Centres, *See Hutton's Mathematics, Banks on Mills, &c.*

CENTRAL FORCES.

1. The quantity of matter in a body is as its magnitude and density; that is, if a body measures 7 cubic feet, and a cubic foot weighs 10 libs, the quantity of matter in that body will be $= 7 \times 10 = 70$ libs.

2. All bodies naturally endeavour to continue in their present state, whether of rest or motion.

3. When a body at rest is struck by a force so as to produce motion, that motion is in proportion to the force, and in the direction of the right line in which it acts.

4. Action and Reaction between any two bodies, are equal and contrary; that is, by Action and Reaction equal changes of motion are produced in bodies acting on each other; and these changes are directed towards opposite or contrary parts.

5. The Momenta, or quantities of motion in moving bodies, are as their masses and velocities; for a body of 20 libs, moving at a velocity of 10 feet, will have a momentum of 200; but a body of 6 libs, moving at the same velocity, will have only 60 for its momentum.

6. A body moving round a central point inclines to fly off in a straight line, from the first impulse of

motion; the force which causes it to leave that line, or move in a circle round the point, is called the Centripetal; and the resistance which it affords, the Centrifugal force; or, in other words, when a body revolves round its centre of motion, the centrifugal force is that power or tendency which the body has to burst or fly asunder; and the centripetal force, is that power which keeps the body from bursting or flying asunder.

It is evident from the last remark, that the greater the velocity, the greater will be the centrifugal force; and from the 5th remark, the greater the mass, the greater the momentum; therefore, as is the weight and velocity of the revolving body, so is the centrifugal force.

Suppose two fly-wheels of the same weight, one of them 12 feet diameter, and revolving in 8 seconds; what must be the diameter of the other, when it revolves in 3 seconds?

The diameter and velocity of the first, must be equal to the diameter and velocity of the second; therefore as $8^2 : 12 : : 3^2$ to the diameter $= \frac{12 \times 3^2}{8^2}$ $= \frac{108}{64} = 1.6875$ foot, the diameter of the second fly-wheel at the circle of percussion.

Again, suppose two fly-wheels of the same diameter, the one revolving in 3 seconds, and the other in 8 seconds; what will be the difference of their weights?

As $3^2 : 8^2$ so is the weight of the one, to the weight of the other.

$\frac{8^2}{3^2} = \frac{64}{9} = 7\frac{1}{9}$, their weights will be to each other as $7\frac{1}{9}$ is to 1; and by knowing the weight of the second, and dividing it by $7\frac{1}{9}$, will give the weight of the first.

In the two preceding Examples, weight and velocity are taken separately.—The following Examples give the centrifugal force, when both weight and velocity are used.

Required the centrifugal force of a fly-wheel, diameter 16 feet, velocity 50 revolutions per minute, and weight $3\frac{1}{2}$ tons?

3.1416 = circumference of a circle the diameter 1.

16 feet = space a body falls through in 1 second of time.

$.8\dot{3}\dot{3}$ = time of one revolution.

$\frac{16 \times 3.1416^2}{16 \times .83\dot{3}^2} = \frac{157.9136}{11.1122} = 14.21$ times the weight in tons, the wt. being $3\frac{1}{2}$ tons; therefore $3.5 \times 14.21 = 49.73$ tons, the centrifugal force.

The stones on which they grind table-knives at Sheffield, are about 44 inches diameter, and weigh about half a ton; the velocity of the circumference is at the rate of 1250 yards in a minute, equal to 326 revolutions; required the centrifugal force?

$22^2 \times 2 = 968$, the square root of which is 31.1 inches, or 2.59 feet, the diameter of the circle of gyration.

As 326 : 60 : : 1 : .184 seconds, the time of one revolution.

$$\frac{2.59 \times 3{,}1416^2}{16 \times .184^2} = \frac{25.5622}{.54169} = 47.18$$ times the weight of the stone: the stone is .5 ton, therefore 47.18 × .5 = 23½ tons centrifugal force.

BANKS.

MOTION, RESISTANCE, AND EFFECT OF MACHINES.

VARIOUS as the modifications of Machines are, and innumerable their different applications; still there are only three distinct objects to which their utility tends.

The first is, in furnishing the means of giving to the moving force the most commodious direction; and when it can be done, of causing its action to be applied immediately to the body to be moved. These can rarely be united; but the former can be accomplished in most cases.

The second, in accommodating the velocity of the work to be performed, to the velocity with which alone a natural power can act.

The third, and most essential advantage of machines, is in augmenting, or rather in modifying the energy of the moving power, in such a manner, that it may produce effects of which it would have been otherwise incapable. For instance, a man might with exertion lift 4 cwt.; but let him apply a lever, and he will lift many times that weight.

The motions produced by machines are of three kinds, viz. Accelerated, Uniform, and Alternate, *i. e.* accelerated and retarded. The first of these always takes place when the moving power is immediately applied; the second, after the machine has been in motion for a short time; the third, in intermitting machines, such as pendulum clocks, &c.; but though a second's pendulum is accelerated the first half second, and retarded the next; still it produces a constant number of vibrations in a given time, and therefore may be considered as a machine of uniform motion.

The grand object in all practical cases, is, to procure a uniform motion, because it produces the greatest effect. All irregularities of motion indicate that there is some point resisting the motion, and to overcome which a part of the propelling power is wasted, and the greatest varying velocity is only equal to that velocity by which the machine would move when its motion is uniform. If the machine moves with an accelerating velocity, it is certain that the power is greater than what balances the opposing resistance, and therefore cannot produce the greatest effect; because the whole resistance is not applied. In both these cases the machine has neither the power nor the effect which it would have if moving uniformly.

When irregularity of motion takes place, particularly in a large heavy machine, it suffers a continual straining and jolting, which must very soon destroy it. It is therefore of the greatest consequence, that, from all machines, every cause tending to produce irregularity of motion should be taken away.

Many fundamental Rules having already been given; and from what has just now been stated respecting the motion of machines, it is thought unnecessary to give any elaborate calculation on the maximum and minimum effects of motion: therefore the following statements of the effects of one horse power, in the different applications enumerated, will conclude this article.

One horse's power, at a maximum, is calculated to lift, by means of a pump, 250 hhds. of water, 10 feet high in one hour.

One horse's power, at a medium, is calculated to drive 100 spindles, with preparation, of cotton yarn twist.

Ditto ditto, 500 spindles, with preparation, of Mule yarn, No. 48.

Ditto ditto, 1000 spindles, with preparation, of Mule yarn, No. 110.—The intermediate numbers in proportion.

Ditto ditto, 12 power looms, with preparation.

STEAM ENGINE.

The Rules of Practical Mechanics, with various Examples, have been stated, embracing the Mechanical Powers, which exhibit the Analysis of Machinery—The Rules for finding the weight and strength of the different parts—The centres of Gravity, Percussion, and Gyration, which determine, according to the nature of the Machine, that point in which all its force is collected—And an explanation of the nature of Motions, and the effect of Machines; showing, that when a Machine is producing the greatest effect, or working at a maximum, that the motion is uniform; the power and resistance being in a proper proportion to each other.

What remains now to be explained, are, the Rules connected with the Steam Engine, Water Wheel, Common and Force Pumps, which are, *viz.*

STEAM ENGINE.

Fuel.—To produce equal heats, $\frac{3}{4}$ cwt. of Newcastle Coal is equal to 1 cwt. of Glasgow Coal, and to $2\frac{1}{4}$ cwt. of Wood, or three times the weight. Also, it takes double the weight of Culm to that of Coal.

Upon the premises of Messrs. Claud Girdwood & Co. of Glasgow, there are two Steam Engines, one of 32 horse power, and one of 12 horse power. The daily consumpt of Culm for the first, upon an average, is 5 waggons, 24 cwt. each; for the second 1½ waggon. Taking an average from these two, gives 34 libs of Culm per hour for each horse power.

The consumpt of Coal per hour for one horse power, is from 13 to 20 libs, according to the strength of the fuel, and the manner by which the furnace is fed.—Consumpt of Wood per hour for one horse power, is 50 libs.—Consumpt of Culm per hour for one horse power, is 34 libs.

I trust I will not be thought impertinent, and, I hope, not partial, in recommending one of the latest, and I may safely say, the best improvement, as yet known, for the economical using of fuel; it is, Brunton of Birmingham's Fire Regulator. This machine feeds the fire in the most regular manner, and nicely proportions the quantity of coal thrown upon the grate, to the quantity of steam required.

Almost the whole of the Public Works, using Steam Engines in London, have this Fire Regulator attached to their Boilers. And from the accounts kept by their Engineers of the quantity of coal consumed, it is found that a saving of from 15 to 25 per cent. is produced by this machine. But the regular manner of feeding the fire, and, consequently, the saving of fuel, are not the only advantages derived from this excellent apparatus. There is no regular fireman needed, the Hopper only requires to be filled with coal in the morning, and no other attendance is necessary; also the supplementary Boiler, which is attached to the large Boiler, gives an additional quantity of Steam, say from 2 till 6 horses, in proportion to the size of the Engine, and preserves the large Boiler from the injurious effects of the fire.

These advantages, which are derived by this Fire Regulator over the usual mode of feeding the fire by hand, make it one of the most useful inventions of the present day, and, in fact, a Steam Engine is not complete without it.

BOILERS—are of various forms, but the most general is proportioned as follows, viz. width 1, depth 1.1, length 2.5; their capacity being, for the most part, two horse more than the power of the Engine for which they are intended.

Boulton and Watt allow 25 cubic feet of space for each horse power, some of the other Engineers allow 5 feet of surface of water.

STEAM—arising from water at the boiling point, is equal to the pressure of the atmosphere, which is in round numbers, 15 libs on the square inch; but to allow for a constant and uniform supply of Steam to the Engine, the safety valve of the Boiler is loaded with 3 libs on each square inch.

The following Table shows the pressure of Steam, according to its heat and expansion.

	Libs.		Degrees.		Times its volume	
Steam predominating over the pressure of the atmosphere, upon a safety valve, if its elastic force be equal to	5	libs ℔ square inch, requires to be maintained by a temperature equal to	227½	degrees of heat by Fahrenheit, and at these respective degrees of heat, steam can expand to about	5	times its volume, and yet continue equal in its elasticity to the pressure of the atmosphere.
	6		230¼		6	
	7		232¾		7	
	8		235¼		8	
	9		237½		9	
	10		239½		10	
	15		250½		15	
	20		259½		20	
	25		267		25	
	30		273		30	
	35		278		35	
	40		282		40	

By small additions to the temperature, an expansive force may be given to steam so as to be equal to 400 times its natural bulk, or in any other proportion, provided the vessels, &c. that contain it be strong in proportion.

The following is a simple Rule for finding the quantity of steam required to raise a given quantity of water to any given temperature.

Rule. Multiply the water to be warmed by the difference of temperature between the cold water and that to which it is to be raised, for a dividend; then to the temperature of the steam add 900 degrees, and from that sum take the required temperature of the water: this last remainder being made a divisor to the above dividend, the quotient will be the quantity of steam in the same terms as the water.

EXAMPLE.

What quantity of steam at 212° will raise 100 gallons of water at 60° up to 212°?

$\frac{212^\circ - 60^\circ \times 100}{212^\circ + 900^\circ - 212^\circ} = 17$ gallons of water formed into steam.

Now, steam at the temperature of 212° is 1800 times its bulk in water; or 1 cubic foot of steam, when its elasticity is equal to 30 inches of mercury, contains 1 cubic inch of water.—Therefore 17 gallons of water converted into steam, occupies a space of 4090¾ cubic feet, having a pressure of 15 libs on the square inch.

Horse Power.—Boulton and Watt suppose a horse able to raise 32,000 libs avoirdupois 1 foot high in a minute.

Desaguliers makes it 27,500 libs.
Smeaton do. 22,916 do.

Boulton and Watt, however, in calculating the power of their Engines, suppose a horse to draw 200 libs at the rate of 2½ miles in a hour, or 220 feet per minute, with a continuance, drawing the weight over a pulley—now, 200 × 220 = 44000, *i. e.* 44000 libs at 1 foot per minute, or 1 lib at 44000 feet per minute.

Length of Stroke.—The stroke of an Engine is equal to one revolution of the crank shaft, therefore double the length of the cylinder. When stating the length of stroke, the length of cylinder is only given, that is, an engine with a 3 feet stroke, has its cylinder 3 feet long, besides an allowance for the piston.

The following Table shows the length of stroke, (or length of cylinder,) and the number of feet the piston travels in a minute, according to the number of strokes the Engine makes when working at maximum.

Length of Stroke.	Number of Strokes.	Feet ⅌ Minute.
Feet 2	43	172
- 3	32	192
- 4	25	200
- 5	21	210
- 6	19	228
- 7	17	238
- 8	15	240
- 9	14	250

CYLINDER.—When the Engine is working at maximum, the effective force of the piston is generally calculated at 10 libs for each square inch of its surface.

To calculate the power of the Engine.

RULE. Multiply the area of cylinder by the effective pressure = 10 libs, the product is the weight the Engine can raise.—Multiply this weight by the number of feet the piston travels in one minute, which will give the momentum, or weight, the Engine can lift 1 foot high per minute; divide this momentum by a horse power, as previously stated, and the quotient will be the number of horse power the Engine is equal to.

EXAMPLE I.

What is the power of an Engine, the cylinder being 42 inches diameter, and stroke 5 feet?

$$\frac{42^2 \times .7854 \times 10 \times 210}{44000} = 66.12 \text{ horse power.}$$

EXAMPLE II.

What size of cylinder will a 60 horse power Engine require, when the stroke is 6 feet?

$$\frac{44000 \times 60}{228 \times 10} = 1158 \text{ inch. area of cylinder.}$$

Note. To find the power to lift a weight at any velocity, multiply the weight in libs by the velocity in feet, and divide by the horse power; the quotient will be the number of horse power required.

Nozles.—The diameter of the valves of Nozles ought to be fully one-fifth of the diameter of cylinder.

Air-Pump.—The solid contents of the Air-Pump is equal to the fourth of the solid contents of cylinder, or when the Air-pump is half the length of the stroke of the Engine, its area is equal to half the area of cylinder.

Condenser—is generally equal in capacity to the Air-pump; but when convenient, it ought to be more; for when large, there is a greater space of vacuum, and the steam is sooner condensed.

Cold Water Pump.—The capacity of the Cold Water Pump depends on the temperature of the water. Many Engines return their water, which cannot be so cold as water newly drawn from a river, well, &c.; but when water is at the common temperature, each horse power requires nearly 7½ gallons per minute.* Taking this quantity as a standard, the size of the pump is easily found by the following Rule, viz.—Multiply the number of horse power by 7½ gallons, and divide by the number of strokes per minute; this will give the quantity of water to be raised each stroke of pump. Multiply this quantity by 231, (the number of cubic inches in a gallon,) and divide by the length of effective stroke of pump, the quotient will be the area.

EXAMPLE.

What diameter of pump is requisite for a 20 horse power Steam Engine, having a 3 feet stroke, the effective stroke of pump to be 15 inches?

$20 \times 7\tfrac{1}{2} = \frac{150}{32} = 4.6875$ gallons the pump lifts each stroke.

$$\frac{4.6875 \times 231}{15} = 72.1875 \text{ inches area of pump.}$$

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* An Engine will work with a less supply of water, say 5 gallons per minute; but when water can be had without a considerable expense of power, 7½ gallons is preferable; because an abundance of water keeps the condenser, &c. cool, and thereby produces a more perfect vacuum.

Hot Water Pump.—The quantity of water raised at each stroke ought to be equal in bulk to the 900th part of the capacity of the cylinder.

Proportions.—The length of stroke being 1, the length of beam to centre will be 2, the length of crank .5, and the length of connecting rod 3.

The following Table shows the force which the the connecting rod has to turn round the crank at different parts of the motion.

Col. A. Decimal proportions of descent of the Piston, the whole descent being 1.

Col. B. Angle between the connecting Rod and Crank.

Col. C. Effective length of the Lever upon which the connecting Rod acts, the whole Crank being 1.

Col. D. Decimal proportions of half a revolution of the Fly-Wheel.

Col. C. Also shows the force which is communicated to the Fly-Wheel, expressed in decimals, the force of the Piston being 1.

A	B	C	D
.0	180°	.0	.0
.05	151½	.46	.128
.10	141	.62	.158
.15	131½	.74	.228
.2	123½	.830	.271
.25	117¼	.892	.308
.3	110¾	.94	.342
.35	104	.976	.377
.4	97½	.986	.41
.45	91¾	1.	.441
.5	85½	1.	.473
.55	80	.986	.507
.6	75	.956	.538
.65	69	.92	.572
.7	62½	.88	.607
.75	57½	.824	.642
.8	49	.746	.68
.85	42	.66	.723
.9	34	.546	.776
.95	23½	.390	.84
1.0	0	.000	1.0

Fly-Wheel—Is used to regulate the motion of the engine, and to bring the crank past its centres. The Rule for finding its weight, is,—Multiply the number of horses' power of the Engine by 2000, and divide by the square of the velocity of the circumference of the wheel per second, the quotient will be the weight in cwts.

EXAMPLE.

Required the weight of a fly-wheel proper for an Engine of 20 horse-power, 18 feet diameter, and making 22 revolutions per minute?

18 feet diameter = 56 feet circumference, × 22 revolutions per minute = 1232 feet, motion per minute ÷ 60 = 20½ feet motion per second; then $20\frac{1}{2}^2 = 420\frac{1}{4}$ the divisor.

20 horse power × 2000 = 40000 dividend.

$$\frac{40000}{420\frac{1}{4}} = 90.4 \text{ cwt. weight of wheel.}$$

Parallel Motion.—The radius and parallel bars are of the same dimensions; their length being generally 1-4th of the length of the beam between the two glands, or one-half of the distance between the fulcrum and gland. Both pairs of straps are the same length between the centres, and which is generally three inches less than the half of the length of stroke. *See Plate 3d.*

Governor, or Double Pendulum.—If the revolutions be the same, whatever be the length of the arms, the balls will revolve in the same plane, and the distance of that plane from the point of suspension, is equal to the length of a pendulum, the vibrations of which will be double the revolutions of the balls. For example; suppose the distance between the point of suspension and plane of revolution be 36 inches, the vibrations that a pendulum of 36 inches will make per minute, is $= \frac{375}{\sqrt{36}} = 62$ vibrations, and $\frac{62}{2} = 31$ revolutions per minute the balls ought to make.

WATER WHEEL.

THIS subject belongs to Hydrodynamics, also the common and force Pumps; and since they are the last of this Treatise, they may be classed under that name, to distinguish them from the preceding subjects in Statics and Dynamics.

WATER. (*Hydrostatics.*)

Hydrostatics is the science which treats of the pressure, or weight, and equilibrium of water, and other fluids, especially those that are non-elastic.

Note 1. The pressure of water at any depth, is as its depth; for the pressure is as the weight, and the weight is as the height.

Note 2. The pressure of water on a surface any how immersed in it, either perpendicular, horizontal, or oblique, is equal to the weight of a column of water, the base being equal to the surface pressed, and the altitude equal to the depth of the centre of gravity, of the *surface pressed*, below the top or surface of the fluid.

PROBLEM I.

In a vessel filled with water, the sides of which are upright and parallel to each other, having the top of the same dimensions as the bottom, the pressure exerted against the bottom, will be equal to the area of the bottom multiplied by the depth of water.

EXAMPLE.

A vessel, 3 feet square and 7 feet deep, is filled with water; what pressure does the bottom support?

$$\frac{3^2 \times 7 \times 1000}{16} = 3937\tfrac{1}{2} \text{ libs Avoirdupois.}$$

PROBLEM II.

A side of any vessel sustains a pressure equal to the area of the side multiplied by half the depth, therefore the sides and bottom of a cubical vessel sustain a pressure equal to three times the weight of water in the vessel.

EXAMPLE I.

The gate of a sluice is 12 feet deep and 20 feet broad; what is the pressure of water against it?

$$\frac{20 \times 12 \times 6 \times 1000}{16} = 90000 = 40\tfrac{1}{4} \text{ tons nearly.}$$

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[illegible]

From Note 2d.—The pressure exerted upon the side of a vessel, of whatever shape it may be, is as the area of the side and centre of gravity below the surface of water.

EXAMPLE II.

What pressure will a board sustain, placed diagonally through a vessel, the side of which is 9 feet deep, and bottom 12 feet by 9 feet?

$\sqrt{12^2 + 9^2} = 15$ feet, the length of diagonal board.

$$\frac{15 \times 9 \times 4\frac{1}{2} \times 1000}{16} = 37969 \text{ libs nearly.}$$

Though the diagonal board bisects the vessel, yet it sustains more than the half of the pressure in the bottom, for the area of bottom is 12 × 9, and the half of the pressure is $\frac{1}{2}60750 = 30375$.

The bottom of a conical or pyramidical vessel sustains a pressure equal to the area of the bottom and depth of water, consequently, the excess of pressure is three times the weight of water in the vessel.

WATER. (*Hydraulics.*)

Hydraulics is that science which treats of fluids considered as in motion, it therefore embraces the phenomena exhibited by water issuing from orifices in reservoirs, projected obliquely, or perpendicularly, in *Jet-d'eaux*, moving in pipes, canals, and rivers, oscillating in waves, or opposing a resistance to the progress of solid bodies.

It would be needless here to go into the minutiæ of hydraulics, particularly when the theory and practice do not agree. It is only the general laws, deduced from experiment, that can be safely employed in the various operations of hydraulic architecture.

Mr. Banks, in his Treatise on Mills, after enumerating a number of experiments on the velocity of flowing water, by several philosophers, as well as his own, takes from thence the following simple rule, which is as near the truth as any that have been stated by other experimentalists.

Rule. Measure the depth (of the vessel, &c.) in feet, extract the square root of that depth, and multiply it by 5.4, which gives the velocity in feet per second; this multiplied by the area of the orifice in feet, gives the number of cubic feet which flows out in one second.

16
5
4

EXAMPLE.

Let a sluice be 10 feet below the surface of the water, its length 4 feet, and open 7 inches; required the quantity of water expended in one second?

$\sqrt{10} = 3.162 \times 5.4 = 17.0748$ feet velocity.

$\frac{4 \times 7}{12} = 2\frac{1}{3}$ feet $\times\ 17.0748 = 39.84$ cubic feet of water per second.

If the area of the orifice is great compared with the head, take the medium depth, and two-thirds of the velocity from that depth, for the velocity.

EXAMPLE.

Given the perpendicular depth of the orifice 2 feet, its horizontal length 4 feet, and its top 1 foot below the surface of water. To find the quantity discharged in one second:

The medium depth is $= 1.5 \times 5.4 = 8.10 - \frac{1}{3} = 5.40 \times 8 = 43.20$ cubic feet.[*]

The quantity of water discharged through slits, or notches, cut in the side of a vessel or dam, and open at the top, will be found by multiplying the velocity at the bottom by the depth, and taking ⅔ of the product for the area; which again multiplied by the breadth of the slit, or notch, gives the quantity of cubic feet discharged in a given time.

EXAMPLE.

Let the depth be 5 inches, and the breadth 6 inches; required the quantity run out in 46 seconds?

The depth is .4166 of a foot.

The breadth is .5 of a foot.

$\sqrt{.4166} = .6455 \times 5.4 \times \frac{2}{3} = 2.3238 \times .4166 = .96825 \times .5 = .48412$ feet per second.

Then $.48412 \times 46. = 22.269$ cubic feet in 46 seconds.

There are two kinds of water wheels, Undershot and Overshot. Undershot, when the water strikes the wheel at, or below the centre. Overshot, when the water falls upon the wheel above the centre.

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[*] The square root of the depth is not taken in this example, but when the depth is considerable, it ought to be taken.

The effect produced by an *undershot* wheel, is from the impetus of the water. The effect produced by an *overshot* wheel, is from the gravity or weight of the water.

Of an undershot wheel, the power is to the effect as 3 : 1.—Of an overshot wheel, the power is to the effect as 3 : 2—which is double the effect of an undershot wheel.

The following is an Abridgement of SMEATON *on* WATER WHEELS.

UNDERSHOT.

Velocity of water in 1″ . = V
Weight of 1 cub. in. of water = W
Area of sluice . . . = A
Quantity of water . . = Q
Power of the water to produce mechanical effect = P

$V.A = Q$ in one second.
$QW.V = P$; Power to produce mechanical effect.

POWER AND EFFECT AT MAXIMUM.

Velocity of Wheel in 1″ . . = v
Effective velocity of water = E
Effect produced by the wheel = e
Weight raised . . . . = w
Velocity of weight raised . = v

$V - v = E$
$wv = e$

$P : e :: 10 : 3.62$ or $3 : 1$
$V : v :: 10 : 3.5$, or $5 : 2$

OVERSHOT.

Descent of water including head and diameter of wheel* = D
The weight of water expended in one second . . . . = W

$D.W = P.$

POWER AND EFFECT AT MAXIMUM.

Power of the water is $= D.W = P$
Effect of the wheel is $= wv = e$

$P : e :: 10 : 6.6$, or $3 : 2$ nearly.
Double that of an Undershot.

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* By Head is understood the distance between the orifice and the part of the wheel on which the water falls. The fall is the perpendicular height from the bottom of the wheel to the orifice.

The velocity at a maximum is = 3 feet in one second.

Since the effect of the overshot is double that of the undershot, it follows that the higher the wheel is in proportion to the whole descent, the greater will be the effect.

The maximum load for an overshot wheel, is that which reduces the circumference of the wheel to its proper velocity, = 3 feet in 1 second; and this will be known, by dividing the effect it ought to produce in a given time, by the space intended to be described by the circumference of the wheel in the same time; the quotient will be the resistance overcome at the circumference of the wheel, and is equal to the load required, the friction and resistance of the machinery included.

The following is an Extract from Banks on Mills, page 152.

" The effect produced by a given stream in falling through a given space, if compared with a weight, will be directly as that space; but if we measure it by the velocity communicated to the wheel, it will be as the square root of the space descended through, agreeably to the laws of falling bodies.

" *Experiment* 1. A given stream is applied to a wheel at the centre; the revolutions per minute are 38.5.

" *Ex.* 2. The same stream applied at the top, turns the same wheel 57 times in a minute.

"If in the first experiment the fall is called 1, in the second it will be 2: then $\sqrt{1} : \sqrt{2} :: 38.5 : 54.4$, which are in the same ratio as the square roots of the spaces fallen through, and near the observed velocity.

"In the following experiments a fly is connected with the water wheel.

"*Ex.* 3. The water is applied at the centre, the wheel revolves 13.03 times in one minute.

"*Ex.* 4. The water is applied at the vertex of the wheel, and it revolves 18.2 times per minute.

"As $13.03 : 18.2 :: \sqrt{1} : \sqrt{2}$ nearly.

"From the above we infer, that the circumferences of wheels of different sizes may move with velocities which are as the square roots of their diameters without disadvantage, compared one with another, the water in all being applied at the top of the wheel; for the velocity of falling water at the bottom or end of the fall is as the time, or as the square root of the space fallen through: for example, let the fall be 4 feet, then, As $\sqrt{16} : 1'' :: \sqrt{4} : \frac{1}{2}''$, the time of falling through 4 feet:—Again, let the fall be 9 feet, then, $\sqrt{16} : 1'' :: \sqrt{9} : \frac{3}{4}''$, and so for any other space, as in the following Table, where it appears that water will fall through one foot in a quarter of a second, through 4 feet in half a second, through 9 feet in 3 quarters of a second, and through 16 feet in one second. And if a wheel 4 feet in diameter moved as fast as the water, it could not revolve in less than 1.5 second, neither could a wheel of 16 feet diameter revolve in less than three seconds; but though it is impossible for a wheel to move as fast as the stream which turns it, yet, if their velocities bear the same ratio to the time of the fall through their diameters, the wheel 16 feet in diameter may move twice as fast as the wheel 4 feet diameter."

TABLE.

Height of the fall in Feet.	Time of falling in Seconds.	Height of the fall in Feet	Time of falling in Seconds.
1	.25	14	.935
2	.352	16	1.
3	.432	20	1.117
4	.5	24	1.22
5	.557	25	1.25
6	.612	30	1.37
7	.666	36	1.5
8	.706	40	1.58
9	.75	45	1.67
10	.79	50	1.76
12	.864		

Power and Effect.—The power water has to produce mechanical effect, is as the quantity and fall of perpendicular height.—The mechanical effect of a wheel is as the quantity of water in the buckets and the velocity.

The power is to the effect as 3 : 2, that is, suppose the power to be 9000, the effect will be

$$=\frac{9000.\times 2}{3}=\frac{18000}{3}=6000.$$

Height of the Wheel.—The higher the wheel is in proportion to the fall, the greater will be the effect, because it depends less upon the impulse, and more upon the gravity of the water; however, the head should be such, that the water will have a greater velocity than the circumference of the wheel; and the velocity that the circumference of the wheel ought to have, being known, the head required to give the water its proper velocity, can easily be known from the rules of Hydrostatics.

VELOCITY OF THE WHEEL.—Banks, in the foregoing quotation, says, "That the circumferences of overshot wheels of different sizes may move with velocities as the square roots of their diameters, without disadvantage."—Smeaton says, "Experience confirms that the velocity of 3 feet per second is applicable to the highest overshot wheels, as well as the lowest; though high wheels may deviate further from this rule, before they will lose their power, by a given aliquot part of the whole, than low ones can be admitted to do; for a 24 feet wheel may move at the rate of 6 feet per second, without losing any considerable part of its power."

It is evident that the velocities of wheels, will be in proportion to the quantity of water and the resistance to be overcome:—if the water flows slowly upon the wheel, more time is required to fill the buckets than if the water flowed rapidly; and whether Smeaton or Banks is taken as a data, the mill-wright can easily calculate the size of his wheel, when the velocity and quantity of water in a given time is known.

EXAMPLE I.

What power is a stream of water equal to, of the following dimensions, viz. 12 inches deep, 22 inches broad; velocity, 70 feet in 11¾ seconds, and fall, 60 feet?—Also, what size of a wheel could be applied to this fall?

$\frac{12 \times 22}{144} = 1.83$ square feet:—area of stream.

11¾″ : 70 : : 60″ : 357.5 lineal feet per min.—velocity.

$357.5 \times 1.83 = 654.225$ cubic feet per minute.

$654.225 \times 62.5 = 40889.0625$ avoir. libs per minute.

$40889.0625 \times 60 = 2453343.7500$ momentum at a fall of 60 feet.

$\frac{2453343.7500}{44000} = 55.7$ horse power.

3 : 2 : : 55.7 : 37.13 effective power.

The diameter of a wheel applicable to this fall, will be 58 feet, allowing one foot below for the water to escape, and one foot above for its free admission.

$58 \times 3.1416 = 182.2128$ circumference of wheel.

$60 \times 6 = 360$ feet per minute, = velocity of wheel.

$\frac{654.225}{360} = 1.8$ sectional area of buckets.

The buckets must only be half full, therefore $1.8 \times 2 = 3.6$ will be the area.

To give sufficient room for the water to fill the buckets, the wheel requires to be 4 feet broad, now, $\frac{3.6}{4} = .9$, say 1 foot depth of shrouding.

$\frac{360}{182.2128} = 1.9$ revolutions per minute the wheel will make.

Power of water . . = 55.7 H. P.
Effective power of do. = 37.13 H. P.
Dimensions of Wheel. Diameter . . . = 58 Feet.
Breadth . . . = 4 Feet.
Depth of shrouding = 1 Foot. *Ans.*

EXAMPLE II.

What is the power of a water wheel, 16 feet diameter, 12 feet wide, and shrouding 15 inches deep.

16 × 3.1416 = 50.2656 circumference of wheel.
12 × 1¼ = 15 square feet, sectional area of buckets.
60 × 4 = 240 lineal feet per minute, = velocity.
240 × 15 = 3600 cubic feet water, when buckets are full; when half full, 1800 cubic feet.
1800 × 62.5 = 112500 avoir. lbs of water per minute.
112500 × 16 = 1800000 momentum, falling 16 feet.
$3 : 2 :: 1800000 : \frac{1200000}{44000} = 27$ horse power.

Buckets.—The number of buckets to a wheel should be as few as possible, to retain the greatest quantity of water; and their mouths only such a width as to admit the requisite quantity of water, and at the same time sufficient room to allow the air to escape.

The Communication of Power.—There are no prime movers of machinery from which power is taken in a greater variety of forms than the water wheel, and among such a number there cannot fail to be many bad applications.

Suffice it here to mention one of the worst, and most generally adopted. For driving a cotton mill in this neighbourhood, there is a water wheel about 12 feet broad, and 20 feet diameter; there is a division in the middle of the buckets upon which the segments are bolted round the wheel, and the power is taken from the vertex: from this erroneous application, a great part of the power is lost; for the weight of water upon the wheel presses against the axle in proportion to the resistance it has to overcome, and if the axle was not a large mass of wood, with very strong iron journals, it could not stand the great strain which is upon it.

The most advantageous part of the wheel, from which the power can be taken, is that point in the circle of gyration horizontal to the centre of the axle; because, taking the power from this part, the whole weight of water in the buckets acts upon the teeth of the wheels; and the axle of the water wheel suffers no strain.

The proper connection of machinery to water wheels is of the first importance, and mismanagement in this particular point is often the cause of the journals and axles giving way, besides a considerable loss of power.

To find the radius of the circle of gyration in a water wheel is therefore of advantage to the saving of power, and the following Example will show the rule by which it is found. *See Centre of Gyration.*

EXAMPLE.

Required the radius of the circle of gyration in a water wheel, 30 feet diameter; the weight of the arms being 12 tons, shrouding 20 tons, and water 15 tons.

30 feet diameter, radius = 15 feet.

$$\left.\begin{array}{l} S\ 20 \times 15^2 = 4500 \times 2 = 9000 \\ A\ \dfrac{12 \times 15^2}{3} = 900 \times 2 = 1800 \end{array}\right\} \text{The opposite side of the water wheel must be taken.}$$

$$\begin{array}{ll} W\ 15 \times 15^2 = 3375 & = 3375 \\ 2 \times \overline{20 + 12} = 64 & \overline{14175} \\ W\ \ \underline{15} & \dfrac{}{79} = 179, \text{ the square root} \\ \ \ 79 & \end{array}$$

of which is 13 $\frac{4}{10}$ feet, the radius of the circle of gyration.

PUMPS.

THERE are two kinds of Pumps, Lifting and Forcing. The Lifting, or Common Pumps, are applied to wells, &c. where the depth does not exceed 32 feet; for beyond this depth they cannot act, because the height that water is forced up into a vacuum, by the pressure of the atmosphere, is about 34 feet.

The Force Pumps are those that are used on all other occasions, and can raise water to any required height.—Bramah's celebrated Pump is one of this description, and shows the amazing power that can be produced by such application, and which arises from the fluid and non-compressible qualities of water.

The power required to raise water any height is equal to the quantity of water discharged in a given time, and the perpendicular height.

EXAMPLE.

Required the power necessary to discharge 175 Ale gallons of water per minute, from a pipe 252 feet high?

One Ale gallon of water weighs $10\frac{1}{4}$ libs avoir. nearly.

$$175 \times 10\tfrac{1}{4} = 1799 \times 252 = \frac{453348}{44000} = 10.3 \text{ horse power.}$$

The following is a very simple Rule, and easily kept in remembrance.

Square the diameter of the pipe in inches, and the product will be the number of libs of water avoirdupois contained in every yard length of the pipe. If the last figure of the product be cut off, or considered a decimal, the remaining figures will give the number of Ale gallons in each yard of pipe; and if the product contains only one figure, it will be tenths of an Ale gallon. The number of Ale gallons multiplied by 282, gives the cubic inches in each yard of pipe; and the contents of a pipe may be found by Proportion.

EXAMPLE.

What quantity of water will be discharged from a pipe 5 inches diameter, 252 feet perpendicular height, the water flowing at the rate of 210 feet per minute?

$$5^2 \times \frac{210}{3} = 175 \text{ Ale gallons per minute.}$$

$$5^2 \times \frac{252}{3} = 2100 \text{ libs water in pipe.}$$

$$\frac{2100 \times 210}{44000} = 10 \text{ horse power required to pump}$$

that quantity of water.

The following Table gives the contents of a pipe one inch diameter, in weight and measure, which serves as a standard for pipes of other diameters, their contents being found by the following Rule.

Multiply the numbers in the following Table against any height, by the square of the diameter of the pipe, and the product will be the number of cubic inches avoirdupois ounces, and Wine gallons of water, that the given pipe will contain.

EXAMPLE.

How many Wine Gallons of water is contained in a pipe 6 inches diameter, and 60 feet long?

$$2.4480 \times 36 = 88.1280 \text{ Wine Gallons.}$$

In a Wine Gallon there are 231 cubic inches.

TABLE.

ONE INCH DIAMETER.

Feet High.	Quantity In Cubic Inches.	Weight in Avoir. Oz.	Gallons Wine Measure.
1	9.42	5.46	.0407
2	18.85	10.92	.0816
3	28.27	16.38	.1224
4	37.70	21.85	.1632
5	47.12	27.31	.2040
6	56.55	32.77	.2448
7	65.97	38.23	.2423
8	75.40	43.69	.3264
9	84.82	49.16	.3671
10	94.25	54.62	.4080
20	188.49	109.24	.8160
30	282.74	163.86	1.2240
40	376.99	218.47	1.6300
50	471.24	273.09	2.0400
60	565.49	327.71	2.4480
70	659.73	382.33	2.8560
80	753.98	436.95	3.2640
90	848.23	491.57	3.6700
100	942.48	546.19	4.0800
200	1884.96	1092.38	8.1600

The resistance arising from the friction of water flowing through pipes, &c. is directly as the velocity of the water, and inversely as the circumference of the pipe.

The data given is a medium, and which is 1-5th of the whole resistance: this is the standard generally adopted, being considered as most correct.

EXAMPLE I.

What is the power requisite to overcome the resistance and friction of a column of water 4 inches diameter, 100 feet high, and flowing at the velocity of 300 feet per minute?

$$\frac{546.19 \times 4^2}{16} = 546.19, \text{ say } 546.2$$

$\frac{546.2 \times 300}{44000} = 3.7$ $\frac{1}{5}$th of which is .7, therefore the power required to overcome the resistance occasioned by the weight and friction of the water will be $3.7 + .7 = 4.4$ H. P., say 4.5 horse power.

EXAMPLE II.

There is a cistern 20 feet square, and 10 feet deep, placed on the top of a tower 60 feet high; what power is requisite to fill this cistern in 30 minutes, and what will be the diameter of the pump, when the length of stroke is 2 feet, and making 40 per minute?

$20 \times 20 \times 10 = 4000$ cubic contents of cistern.

$\frac{4000}{30} = 133.3$ cubic feet of water per min.

$\frac{133.3 \times 1000}{16} = 8331.25$ libs avoir. per minute.

$\frac{8331.25 \times 60}{44000} = 11.36$ horse power, 1-5th of which is $= 2.27 + 11.11 = 13.63$ horse power required.

$\frac{133.3}{2 \times 40 = 80} = 1.7 \times 144 = \frac{244.80}{.7854} = 311.7$, now

$\sqrt{311.7} = 17.6$ inches, diameter of pump required.

Founders generally prove the pipes they cast to stand a certain pressure, which is calculated by the weight of a perpendicular column of water, the area being equal to the area of the pipe, and the height equal to any given height.

To ascertain the exact pressure of water to which a pipe is subjected, a safety-valve is used, generally of 1 inch diameter, and loaded with a weight equal to the pressure required: for example, a pipe requires to stand a pressure of 300 feet, what weight will be required to load the safety-valve one inch diameter?

$$\overset{\text{Feet.}}{300} \times 12 = \overset{\text{Inches.}}{3600} \times .7854 = \frac{2827.4400 \times 1000}{1728} = \frac{\overset{\text{Ounces.}}{1636\frac{1}{4}}}{16}$$

= 102 libs 4¼ oz. weight required.

Each of the weights for the safety-valves of these Hydrostatic proving-machines are generally made equal to a pressure of a column of water 50 feet high, the area being the area of the valve.

50 feet of pressure on a valve 1 inch diam. = 17.06 libs
50 do. do. do. 1¼ do. = 26.65 do.
50 do. do. do. 1½ do. = 38.38 do.
50 do. do. do. 2 do. = 68.24 do.

In pumping, there is always a deficiency owing to the escape of water through the valves; to account for this loss, there is an allowance of 3 inches for each stroke of piston rod: for example, a 3 feet stroke may be calculated at 2 feet 9 inches.

There is a town, the inhabitants of which amount to 12000, and it is proposed to supply it with water, from a river running through the low grounds 250 perpendicular feet below the best situation from the reservoir.

It is required to know the power of an Engine capable of lifting a sufficient quantity of water, the daily supply being calculated at 10 Ale gallons to each individual: also, what size of pump and pipes are requisite for such?

$12000 \times 10 = 120000$ gallons per day.

Engine is to work 12 hours, $\frac{120000}{12} = 10000$ gallons per hour.

$$\frac{10000}{60} = 166.6 \text{ gallons per minute.}$$

The pump to have an effective stroke of $3\frac{3}{4}$ feet, and making 30 strokes per minute.

$$\frac{166.6}{30} = 5.5533 \text{ gallons each stroke.}$$

$282 \times 5.6 = 1579.2$ cubic inches of water each stroke.

$$\frac{1579.2}{3 \text{ feet } 9 \text{ in} = 45 \text{ in.}} = 35.1 \text{ inches, area of pump.}$$

$$\frac{35.1}{.7854} = 44.7, \text{ therefore } \sqrt{44.7} = 6.7 \text{ diam. of pump.}$$

The pipes will require to be at least the diameter of the pump; if they are a little more, the water will not require to flow so quickly through them, and thereby cause less friction.

The power of the Engine will be

166.6 gall. $\times\ 10\frac{1}{4}$ lb $\times\ 250$ feet $= 426925$ momentum.

$\frac{426925}{44000} = 9.7$, add 1-5th $= 11.64$ horse power.

$\frac{426925}{32000} = 13.3$, —— $= 15.96$ do. *Watt.*

$\frac{426925}{27500} = 15.5$, —— $= 18.6$ do. *Desaguliers.*

$\frac{426925}{22916} = 18.6$, —— $= 22.32$ do. *Smeaton.*

MISCELLANIES.

Required the weight of a cast iron ball of 3 inches diameter, supposing the weight of a cubic inch of the metal to be 0.258 libs avoirdupois? HUTTON.

$3^3 \times .5236 \times .258 = 3.6473976$ libs avoir.

Required the weight of a hollow spherical iron shell, 5 inches in diameter, the thickness of the metal being 1 inch? HUTTON.

$5^3 \times .5236 \times .258 = 16.88610 - 3.6473976 = 13.23871$ libs avoirdupois.

It is proposed to determine the proportional quantities of matter in the earth and moon; the density of the former being to that of the latter, as 10 to 7, and their diameters, as 7930 to 2160. HUTTON.

$\frac{7930^3 \times 10}{2160^3 \times 7} = 71$ nearly: that is, 71 to 1 nearly.

There are two bodies, of which the one contains 25 times the matter of the other, or is 25 times heavier; but the less moves with 1000 times the velocity of the greater; in what proportion then are the momenta, or forces with which they move? HUTTON.

$\frac{1000}{25} = 40$; that is, the less moves with a force 40 times greater.

It is proposed to divide the Beam of a Steel-yard, or to find the points of division where the weights of 1 2 3 4, &c. libs on the one side, will just balance a constant weight of 95 libs, at the distance of 2 inches on the other side of the fulcrum, the weight of the Beam being 10 libs, and its whole length 36 inches.

$36 : 10 :: 2 : 55 = \frac{20.000}{36} = .55$ weight of short arm.

$2 =$ length of short arm.

$36 - 2 = 34 =$ length of long arm.

$10 - .55 = 9.45 =$ weight of long arm.

$95 \times 2 = 190$ momentum of weight at end of short arm.

$.55 \times 1 = .55$ do. of short arm.

190.55 whole momentum of weight and arm.

$\frac{9.45 \times 34}{2} = 160.65$ momentum of long arm.

$190.55 - 160.65 = 29.90$, or 30, the excess of momentum.

$\frac{30}{1} = 30$ inches from the fulcrum for the one lib weight.

$\frac{30}{2} = 15$ . . . . two do.

$\frac{30}{3} = 10$ . . . . three do.

$\frac{30}{4} = 7\frac{1}{2}$ . . . . four do.

How long after firing the Tower guns, may the report be heard at Shooter's-Hill, supposing the distance to be 8 miles in a straight line?

$$8 \times \frac{14}{3} = 8 \times 14 = \frac{112}{3} = 37\tfrac{1}{3} \text{ seconds.}$$

If a lever, 40 effective inches long, will, by a certain power thrown successively on it, in 13 hours, raise a weight 104 feet; in what time will two other levers, each 18 effective inches long, raise an equal weight 73 feet?

If 40 : 104 : : 18 : 46.8, one of the 18 inch levers would raise in 13 hours.

If 46.8 × 2 = 93.6 : 13 : : 73 : 10 hours 8⅓ minutes, time to raise the weight 73 feet.

There are two bodies, the one of which weighs 100 libs, the other 60 libs; but the lightest is impelled by a force 8 times greater than the other; the proportion of the velocities with which these bodies move, is required? HUTTON.

$\frac{100 \times 8 \times 3}{60} = 40$, that is, the velocity of the greater is to that of the less, as 3 to 40.

Supposing one body to move 30 times swifter than another; as also the swifter to move 12 minutes, the other only 1; what difference will there be between the spaces described by them, supposing the last has moved 5 feet?

30 × 5 = 150 feet in first minute. 150 × 12 = 1800 — 5 = 1795 feet difference between the spaces described.

If a Stone be 19½ seconds in descending from the top of a precipice to the bottom, what is its height?

$$19.5^2 = 380.25 \times 16\tfrac{1}{12} = 6115.6875 \text{ feet.}$$

It is proposed to determine the length of a Pendulum vibrating seconds, in the latitude of London, where a heavy body falls through $16\frac{1}{12}$ feet in the first second of time?

$$\left.\begin{array}{l}3.1416 \text{ circumference, the diameter being } 1.\\ 16\frac{1}{12} \text{ feet} = 193 \text{ inches fall in the } 1'' \text{ of time.}\end{array}\right\}*$$

$$\begin{array}{r}193 \times 2 =\\ 3.1416^2 =\end{array}\frac{386.00000000}{9.86965056} = 39.109 \text{ inches,}$$

or 39.11 inches.

By experiment this length is found to be $39\frac{1}{8}$ inches.

What is the length of a Pendulum vibrating in 2 seconds, and another in half a second?

$\sqrt{39\frac{1}{8}} = 6.25 \times 60 = 375.$ (*See Art. Pendulum.*)

$\frac{375}{30} = 12.5$ squared $= 156.25$ inches the length of a 2 seconds' Pendulum.

$\frac{375}{120} = 3.125$ squared $= 9.765625$ inches the length of a $\frac{1}{2}$ second's Pendulum.

---

* When a Pendulum vibrates in a Cycloid: the time of one vibration, is to the time a body falls through half the length of the Pendulum, as the circumference of a circle, is to its diameter.—The times in which bodies descend through similar parts of similar curves are as the square roots of their lengths. Therefore $\sqrt{\frac{1}{2}l} : \sqrt{g} :: 1'' : \sqrt{\frac{2g}{l}}$ l being = length and g equal to the fall in $1'$ now as $1 : 3.1416 :: \sqrt{\frac{2g}{l}} : 3.1416 \times \sqrt{\frac{2g}{l}}$ let $3.1416 = p$ then $p^2 = \frac{2g}{l}$. $\frac{1}{2}p^2 \times l = g$. $\frac{g}{\frac{1}{2}p^2} = l$

There are two bodies, the one of which has passed over 50 miles, the other only 5; and the first had moved with 5 times the celerity of the second; what is the ratio of the times they have been in describing those spaces?

25 velocity of first.
5 do. of second.

$$\frac{\text{Space } 5}{\text{Velocity } 5} = 1 \text{ time of the second.}$$

$$\frac{\text{Space } 50}{\text{Velocity } 25} = 2 \text{ time of the second;}$$

therefore the ratio of the times will be 2 to 1.

Counted 17″ between the time of seeing the flash, and hearing the report of a gun, what is the distance from it?

Sound flies through the air uniformly at the rate of about 1142 feet in 1 second of time, or a mile in $4\frac{2}{3}$ or $\frac{14}{3}$ seconds.

$$\frac{1142 \times 17}{5280} = \overset{\text{M.}}{3} \,.\, \overset{\text{Feet.}}{3574}, \text{ or } 3\tfrac{2}{3} \text{ miles distance.}$$

How far off was the cloud from which thunder issued, whose report was 5 pulsations in the wrist, after the flash of lightning, counting 75 pulsations to a minute?

$$\frac{60 \times 5}{75} = 4 \times 1142 = 4568 \text{ feet, or } 1522\tfrac{2}{3} \text{ yards.}$$

PROBLEM.

To determine how far a man, who pushes with the force of 100 *libs, can thrust a sponge into a piece of ordnance, whose diameter is* 5 *inches, and length* 10 *feet, when the barometer stands at* 30 *inches: the vent, or touch-hole, being stopped, and the sponge having no windage, that is, fitting the bore quite close?*

A column of quicksilver 30 inches high, and 5 inches diameter, is $5^2 \times 30 \times .7854 = 589.05$ inches; which, at 8.102 oz. each inch, weighs, 4772.48 oz. or 298.28 libs, which is the pressure of the atmosphere alone, being equal to the elasticity of the air in its natural state; to this adding the 100 libs, gives 398.28 libs, the whole external pressure. Then, as the spaces which a quantity of air possesses under different pressures, are in the reciprocal ratio of those pressures, it will be, as 398.28 : 298.28 : : 10 feet or 120 inches : 90 inches nearly, the space occupied by the air; therefore $120 - 90 = 30$ inches, is the distance sought.

From this Problem of Dr. Hutton's the following formula is easily obtained, *viz.*

a = whole length of tube, or volume of air in its natural state.
b = pressure of air in its natural state.
d = space occupied by the compressed air.
n = distance the air is compressed.
x = force exerted to compress the air or the pressure of the air when compressed.

$$b + x : b :: a : d$$

$$a = \frac{(b + x)\,d}{b} \Big\} \qquad d = \frac{a\,b}{b + x} \Big\}$$

$$b = \frac{x\,d}{a - d} \Big\} \qquad x = \frac{a\,b}{d} - b \Big\}$$

$$n = a - d$$

The following Table shows the force of air, when compressed at — times, its volume calculated according to the preceding Theorem.

Times the volume	Pressure on the square inch		Times the volume	Pressure on the square inch.		Times the volume	Pressure on the square inch.	
	Libs.	*Oz.*		*Libs.*	*Oz.*		*Libs.*	*Oz.*
2	15	3	18	258	3	34	501	3
4	45	9	20	288	9	36	531	9
6	75	15	22	318	15	38	561	15
8	106	5	24	349	5	40	592	5
10	136	11	26	379	11	42	622	11
12	167	1	28	410	1	44	653	1
14	197	7	30	440	7	46	683	7
16	227	13	32	470	13	48	713	13

To find the pressure of air at any number of volumes, is very easy and simple—Multiply the pressure of the air, say 15 libs on the square inch, by the number of volumes required, then deduct one volume for the circumambient atmosphere, or multiply 15 libs by one less than the number of volumes or atmospheres required.

TABLE

OF THE PROPERTIES OF VARIOUS BODIES.

BODIES.	Specific Gravity.	Weight of a cubic foot.	Will bear on a square inch without permanent alteration.	Melts at degrees.	Cohesive force of a square inch.	Crushed by a force on square inch.	Absorbs of its weight of water.
WOODS.		*Libs.*	*Libs.*				
Ash .	0.75	47.5	3540				
Beech	0.696	45.3	2360				
Elm	0.544	34.	3240				
Yellow & Red Fir .	0.557	34.8	4290				
White do. .	0.47	29.3	3630				
Mahogany	0.56	35.	3800				
English Oak	0.83	52.	3960				
Americ. Yellow Pine	0.46	26.75	3900				
METALS.							
Cast Brass ...	8.1	506.25	- - -	1869°	18000		
Cast Iron	7.207	450.	15.300	3479°	- - -	93000	
Copper..	8.75	549.	- - -	2548°	33000		
Malleable Iron.....	7.6	475.	17800				
Hammer'd do.	....	487.					
Lead....	11.352	709.5	1500	612°			
Steel....	7.84	490.	- - -	- - -	130000		
Cast Tin	7.291	455.7	2880	442°			
Cast Zinc	7.028	439.25	5700	648°			
STONE, &c.							
Brick ...	1.841	115.	- - -	- - -	275	562	.066
Chalk...	2.315	144.7	- - -	- - -	- - -	500	
Clay	2.	125.					
Aberdeen Granite.	2.625	164.	- - -	- - -	- - -	10910	
White Marble .	2.706	169.	- - -	- - -	1811	6060	
Porphyry	2.871	179.	- - -	- - -	- - -	35568	
Welsh Slate....	2.752	172.	- - -	- - -	12800		
Portland Stone ...	2.113	132.	- - -	- - -	857	3729	.0625
Bath do.	1.975	123.4	- - -	- - -	478	- - -	.077
Craigleith do.	2.362	147.6	- - -	- - -	772	5490	.0158
Dundee do.	2.621	163.8	- - -	- - -	2661	6630	.002

TABLE OF THE WEIGHT OF CAST IRON PIPES.

Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.		
1	¼	3ft6	0	0	12	6½	⅝	9	3	2	21	11½	¾	9	7	2	8
	⅜	3ft6	0	0	21		¾	9	4	1	21		1	9	10	1	2
1½	¼	4ft6	0	0	21		1	9	6	0	14	12	½	9	5	0	24
	⅜	4ft6	0	1	4	7	⅜	9	2	1	7		⅝	9	6	2	8
2	¼	6	0	1	8		½	9	3	0	7		¾	9	7	3	20
	⅜	6	0	2	0		⅝	9	3	3	20		1	9	10	3	0
2½	¼	6	0	1	16		¾	9	4	3	5	12½	½	9	5	1	16
	⅜	6	0	2	10		1	9	6	2	4		⅝	9	6	3	9
	½	6	0	3	10	7½	⅜	9	2	2	4		¾	9	8	1	0
3	¼	9	0	2	20		½	9	3	1	6		1	9	11	0	21
	⅜	9	1	0	6		⅝	9	4	0	22	13	½	9	5	2	20
	½	9	1	1	12		¾	9	5	0	10		⅝	9	7	0	14
	⅝	9	1	3	6		1	9	7	0	0		¾	9	8	2	7
	¾	9	2	1	0	8	½	9	3	2	4		1	9	11	2	12
3½	¼	9	0	3	0		⅝	9	4	1	25	13½	½	9	5	3	7
	⅜	9	1	0	21		¾	9	5	1	18		⅝	9	7	1	12
	½	9	1	2	14		1	9	7	1	16		¾	9	8	3	16
	⅝	9	2	0	8	8½	½	9	3	3	2		1	9	11	3	24
	¾	9	2	2	0		⅝	9	4	2	26	14	½	9	6	0	4
4	⅜	9	1	1	10		¾	9	5	2	22		⅝	9	7	2	16
	½	9	1	3	12		1	9	7	3	8		¾	9	9	1	0
	⅝	9	2	1	12	9	½	9	4	0	0		1	9	12	1	14
	¾	9	2	3	21		⅝	9	5	0	4	14½	½	9	6	0	24
4½	⅜	9	1	2	2		¾	9	6	0	2		⅝	9	7	3	14
	½	9	2	0	4		1	9	8	0	26		¾	9	9	2	2
	⅝	9	2	2	14	9½	½	9	4	0	18		1	9	12	3	6
	¾	9	3	0	21		⅝	9	5	1	0	15	½	9	6	1	21
5	⅜	9	1	2	22		¾	9	6	1	6		⅝	9	8	0	14
	½	9	2	1	10		1	9	8	2	20		¾	9	9	3	7
	⅝	9	2	3	17	10	½	9	4	1	10		1	9	13	0	26
	¾	9	3	1	24		⅝	9	5	1	26		1¼	9	16	3	5
5½	⅜	9	1	3	10		¾	9	6	2	14	15½	½	9	6	2	14
	½	9	2	2	0		1	9	9	0	8		⅝	9	8	1	14
	⅝	9	3	0	18	10½	½	9	4	2	14		¾	9	10	0	10
	¾	9	3	3	7		⅝	9	5	3	7		1	9	13	2	17
	1	9	5	0	12		¾	9	7	0	0		1¼	9	17	1	6
6	⅜	9	2	0	0		1	9	9	2	0	16	½	9	7	0	22
	½	9	2	2	21	11	½	9	4	3	14		⅝	9	8	3	7
	⅝	9	3	1	17		⅝	9	6	0	11		¾	9	10	1	20
	¾	9	4	0	16		¾	9	7	1	7		1	9	14	0	8
	1	9	5	2	20		1	9	9	3	20		1¼	9	17	3	14
6½	⅜	9	2	0	16	11½	½	9	5	0	7		1½	9	21	3	4
	½	9	2	3	20		⅝	9	6	1	12		2	9	29	3	21

The foregoing Table of the weight of cast iron pipes, gives the length of pipe according to the diameter of bore, as generally used in practice.

Diameter of bore in inches.
Thickness of metal in inches
Length of pipe in feet.

It is found to be of great use in making out estimates of pipes:—for instance, it is required to know the weight of a range of pipes 225 feet long, 7½ inches diameter of bore, and metal $\frac{5}{8}$ths of an inch thick.

9)225
25 pipes in the whole length.

One pipe weighs 4 . 0 . 22, which multiplied by 25, is equal to 104 . 3 . 18, or 5 tons, 4 cwt. 3 quarters, 18 libs, weight of the whole range.

The following is a Table of the velocity of Motion, for boring cast iron cylinders, pumps, &c. and heavy Turning, with fixed cutters.

It will be observed, that the surface bored is constantly the same, 78.54 feet per minute; this velocity is found to be the most advantageous: a velocity greater than this, not only takes the temper out of the cutters, but also causing more heat, expands the metal; and if the machine stops but for a few seconds, a mark is left from the contraction of the metal.

Turning has a velocity double to that of boring.

TABLE.

BORING.		TURNING.	
Inches diameter	Revolutions of Bar ⅌ Minute.	Inches diameter	Revolutions of Shaft ⅌ Minute
1	25.	1	50.
2	12.5	2	25.
3	8.33	3	16.67
4	6.25	4	12.50
5	5.	5	10.
6	4.16	6	8.32
7	3.57	7	7.15
8	3.125	8	6.25
9	2.77	9	5.55
10	2.5	10	5.
15	1.66	15	3.33
20	1.25	20	2.50
25	1.	25	2.
30	0.833	30	1.667
35	0.714	35	1.430
40	0.625	40	1.250
45	0.56	45	1.12
50	0.5	50	1.
60	0.417	60	0.834
70	0.358	70	0.716
80	0.313	80	0.626
90	0.278	90	0.556
100	0.25	100	0.50

N. B. The progression of the cutters may be 1-16th of an inch for the first cut, and for the last 1-24th.

If hand-tools are employed in turning, the velocity may be considerably increased.

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